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APPLICATION OF BINOMIAL DISTRIBUTION IN LOGISTICS SYSTEMS

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Abstract: *A downtime interval during which a machine is performing no work due to lack of a spare part is an important economical issue for companies. It has to know in advance the need for spare elements to ensure reliable operation of the machines. In order to determine the required amount of spare elements used a scientific approach. In this paper is proposed a well-known statistical approach to inventory management. The binomial distribution permits to analyze without difficulties the operational reliability and calculating the spare parts demand. Some of results are given by engineering software.*

Key words: *statistical calculations, spare elements.*

I. Introduction.

Replacement elements are a necessary part of the maintenance of industrial production machinery, tools and equipment. A downtime interval during which a machine is performing no work due to lack of a spare part leads to failure to comply with the accepted orders or the production program and has a number of negative consequences. On the other hand, the effort to maintain continuous performance leads to excess inventory. As a result, companies and organizations have to pay additional financial resources, which in itself do not bring revenue. Globally, the accumulation of surplus spare parts also increases with the development of industrial society. According to Bancorp Financial Services, for example, the annual expenses of spare parts inventory management are over 700 billion dollars, which makes up about 8 percent of Gross Domestic Product of the United States, Jasper, 2006 [4]. Therefore, it is necessary to seek methods to reduce the unproductive capital of the ever-rising inventories in the industry.

The scientific approach in determining the optimal amounts of spare components requires the application of the theory of reliability, which aims to ensure reliable operation of the machines. Logistics system input data is determined by the mathematical apparatus of the theory. Thus the defective item for a machine is delivered just in the moment of failure. By *failure* is meant the condition of the item, where it is not possible to carry out the assigned functions, in other words failure here is a malfunction. In solving engineering problems it is often recommended by the theory of inventory management to use complex mathematical expressions. These achieve correct results, but significant time is lost and additional resources are consumed. In most cases, it appears that in the implementation of the current tasks, simple mathematical equations with one to two parameters can be applied with sufficient precision.

In the classical case, in order to determine the amount of spare components for a machine it is necessary to calculate the probabilities of exactly ***k*** details are to perform failure. The formulated problem can be solved statistically in several ways. One of them, which do not require extensive preparation and experimental complex calculations, is to use discrete binomial random variable X.

The purpose of this report is to analyze the possibilities of application of the discrete binomial random variable X in determining the need for spare parts that help to maintain the operational reliability of the machines.

II. A model for determining the amount of identical replacement parts.

A population with a volume of *N* identical elements contains *M* inoperable and *N-M* operable details. The probability of event *A* – the detail is operable is

(1) $$P(A)=\frac{M}{N}=p,$$

and the probability of the event $\overline{A}$ – the detail is inoperable is

(2) $$P(\overline{A})=\frac{N-M}{N}=q$$

It is evident that $p+q=1$.

To meet the terms of the binomial distribution p should be $p=const$, where the research element returns to the population. Engineering practice cannot afford such a situation, but due to the elementary mathematical apparatus and reasonable accuracy the model is widely used in repeatable amounts of details with volume n.

Let $P(X=m)=p_n(m)$ be the probability that the quantities of *n* details the event (case) *X* = m - inoperable details will appear *m* times. For the determination of the probability should be analyzed the cases of the occurrence of the events *A* and $\overline{A}$ in

(3) $$p(A_1)=p(A_2)=p \text{ and } p(\overline{A_1})=p(\overline{A_2})=q,$$

in which *p* – probability of occurrence of event A;

q – probability of occurrence of event $\overline{A}$.

After applying the theorem of multiplication of independent events is obtained the formula for finding the probability of an event $p_n(m)$ [1, 2]

(4) $$p_n(m) = C_n^m p^m (1-p)^{n-m}$$

in which C_n^m – binomial coefficient takes values from *n* details by *m*; $m = 0, 1, 2, \ldots, n$.

Binomial coefficient depends on the number of combinations consisting of *n* detail of *m*-grade. The coefficient is obtained by the decomposition of Newton's Binomial series and is given by the formula [5]

$$C_n^m = \binom{n}{m} = \frac{n!}{m!(n-m)!}$$

Discrete distribution function of the random quantity *X* of inoperable details has the type [2]

(5) $$F(x) = P(X \leq x) = \\ = \sum_{m=0}^{x} \binom{n}{m} p^m (1-p)^{n-m}.$$

The distribution function (5) indicates the probability that the amount of *n* identical machine details contains no more than *m* inoperable details [8]

(6) $$P(0 \leq X \leq x) = P_n(x) = \sum_{m=0}^{x} \binom{n}{m} p_n(x).$$

In the context of the problem in the expression (6) may be found the probability of occurrence of the event "the detail is inoperable" from m_1 to m_2 times, considering the amount of the identical machine details – *n*

(7) $$P(m_1 \leq X \leq m_2) = P_n(m_1, m_2) = \\ = \sum_{m=m_1}^{m_2} \binom{n}{m} p^m (1-p)^{n-m}.$$

The next step, according to the Statistical Decision Theory, is to determine the cumulative distribution function for calculating the parameter of the binomial distribution. One of the best known methods commonly used for obtaining the point estimate of the parameter is the maximum likelihood method, which achieves satisfactory asymptotic properties of the calculated numerical values [6]. For this purpose is formulated the log-likelihood function (logarithmic function of likelihood)

(8) $$\Lambda(x_1, \ldots, x_k, p) = \ln L(x_1, \ldots, x_k, p) = \\ = \ln \binom{n}{m} + m \ln p + (n-m).\ln(1-p),$$

where in *L* is the likelihood function [8].

At the time of occurrence of $X=m$ inoperable elements most reliable will be that p_* value of the parameter *p* for which the probabilities $P(X=m)$ are as large as possible (closest to 1). Therefore, as an estimate of the parameter *p* is assumed the p_* value, for which

$$L(x_1,\ldots,x_k,p)=max\ ,$$

then p_* can be determined from the equation

$$\frac{dL}{dp}=0 \quad \text{или} \quad \frac{d\Lambda}{dp}=0\,.$$

After the differentiation of the logarithmic likelihood function of the binomial distribution for the point estimate of the parameter p is obtained

(9) $$p^*=\frac{x}{n}\,.$$

Since there is some ambiguity in the point estimate of the parameter, although in this case there are certain properties available, like Unbiasedness, Consistency and Sufficiency, it is necessary to implement the Interval evaluation method. For the probability that the amount of n identical machine details contains no more than m inoperable details (10) a confidence interval is defined. Let α specify the significance level, then the probability confidence limits are

(10) $$p_{\alpha/2}\le p^*\le p_{1-\alpha/2}\,.$$

The limits of the confidence interval are calculated using the relationship between the function of the binomial distribution and Fisher-Snedecor or F-distribution [7]

(11) $$p_{\alpha/2}=\frac{x}{x+(n-x+1)F(\alpha/2,k_1,k_2)}=\frac{m}{m+(n-m+1)F(\alpha/2,k_1,k_2)},$$

(12) $$p_{1-\alpha/2}=1-\frac{n-x}{n-x+(x+1)F(\alpha/2,k_3,k_4)}=\frac{m+1}{n-m+(m+1)F(1-\alpha/2,k_1,k_2)}$$ in which $k1=2m$; $k2=2(n-m+1)$

$k3=2(m+1)$; $k4=2(n-m)$.

The point estimate of the parameters p and n is given by the formulas [3]

$$p^*=1-\frac{d^2}{\bar{x}} \quad \text{и} \quad n^*=\frac{\bar{x}}{p^*},$$

in which $\bar{x}$ is the arithmetic mean of the value for the inoperable identical elements;

d^2 corrected dispersion.

The formulas for the arithmetic mean and the corrected dispersion have the following type [5]

$$\bar{x} = \frac{\sum_{m=0}^{k} m\nu_m}{\sum_{m=0}^{k} \nu_m} \quad \text{и} \quad d^2 = \frac{\sum_{m=0}^{k} \left(m - \bar{x}\right)^2 \nu_m}{\sum_{m=0}^{k} \nu_m - 1},$$

in which v_m is the absolute frequency of the numerical value of inoperable elements.

The calculated value of n_* is rounded to the nearest integer. With the result in formula (10) is recalculated the value of parameter p_*.

The statistical evaluation of the technical state of the elements of the machines allows checking whether the binomial distribution is appropriately chosen for the presentation of the empirical distribution. At $p_*<0$, the binomial distribution does not correspond to the statistical data about failure details.

The procedure to test the hypothesis concerning the quantity of inoperable elements in a specified nomenclature of n identical elements considers the estimated probability $p_H=x_H/n$ in which the null hypothesis is

$$H_0: p=p_H,$$

and the alternative hypothesis is

$$H_1: p \neq p_H,$$

includes: 1) setting up a confidence interval in expressions (11) и (12) and 2) definition of p_* in expression (10). If the calculated p_* is within the confidence interval - the null hypothesis is accepted. If not, it is rejected and the alternative hypothesis is accepted.

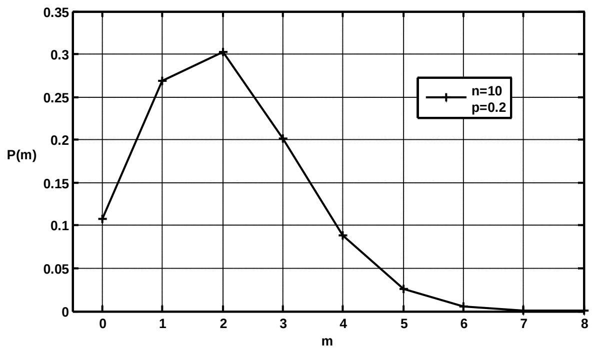

Figure 1.

The model of determining the amount of identical replacement parts is accomplished by MATLAB software for engineering. The behaviour of the function of the distribution depending on the variation of the parameters of the model is shown in figure 1, figure 2 and figure 3.

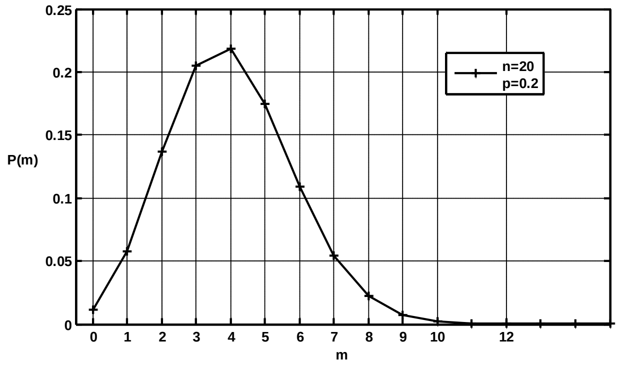

Figure 2.

The model is two-parametric and given a probability of failure and a number of identical parts, it is possible to predict the need for replacement elements.

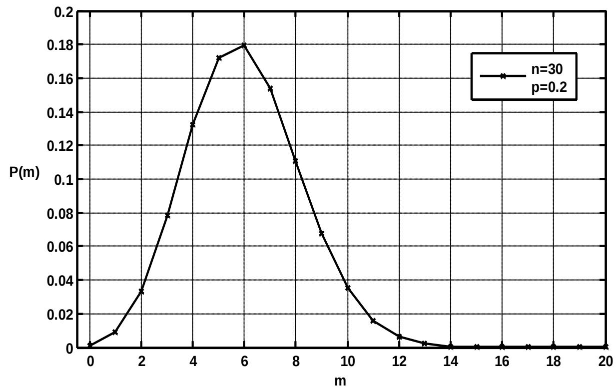

Figure 3 .

As can be seen from the graphs, with the increase of the amount of elements to be reserved, the binomial distribution approximates to normal.

III. Conclusion

The binomial distribution of inoperable elements of random quantity is appropriate for calculating the probability of failure when the system uses partial surplus. It permits to analyze without difficulties the operational reliability of the machines within certain limits and thus to create forecasting methods for calculating the spare parts demand.

Due to the simplicity of the mathematical apparatus, using the binomial distribution leads to a significant time reduction needed for the defective parts statistical data processing. Consequently, the machines downtime is shortened and is achieved economic impact of proper management of stocks of spare parts.

The model for determining the amount of identical replacement parts can be successfully applied in logistics support of industrial machinery.

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