Journal scientific and applied research, vol. 7, 2015
Association Scientific and Applied Research
International Journal

Original Contribution

ISSN 1314-6289

MATHEMATICAL METHODS FOR DECISION-MAKING

Donika Dimanova

*KONSTANTIN PRESLAVSKY UNIVERSITY OF SHUMEN, SHUMEN 9712,
115, UNIVERSITETSKA STR.*
E-mail: d.dimanova@gmail.com

Abstract: The report will set out a systematic introduction to the mathematical methods that facilitate decision-making in many areas of human activity. Especially important are these methods in solving problems in the management and decision-making in emergency situations. The formation of strategic and tactical decisions, the governing body must take into account the multiple and conflicting considerations of which to seek the most effective option.

KEYWORDS: *modeling, Mathematical models, Statistical evaluations, decision-making, emergency situations*

Introduction

An important part of the work of executive bodies is quick and timely response to natural disasters. At formation of strategic and tactical decisions, the control body must take into account the multiple and conflicting considerations of which to seek an option, which under the existing circumstances is most effective. This most effective option for management decision making is called optimal. Therefore, if there are several options for building management strategies we must consider all possible solutions and choose the best among them, which is called an optimal solution. In terms of the development of the emergency situation, the examination of all possible options is virtually impossible. This necessitates the use of modeling and simulations of disaster situations, which is a prerequisite for making science-based decisions in the management [1].

In this regard the purpose of the report is as follows:

1. To systematize the mathematical methods which support science-based management decisions.

2. To characterize the structure of the studied processes by different signs in order to find the best solution.

One of the most important tasks in the study is the construction of models of the studied objects. The activity in the construction, analysis and verification of the model is called modeling. A basic concept in modeling is the term "model". It means a simplified system with material substance or abstract nature, which reflects only separate but important properties of the studied object, called the "original" [1]. Often when we say the word "model" we want to emphasize the difference between the real, objectively existing world and an imaginary abstract model world, which is a product of our mind, and that "exists" in the form of statements, formulas, mathematical symbols and connections and schemes, and some other means.

There are several types of modeling that can be grouped into two major types [2]:

- Materially-physical modeling - the model is implemented as a physical object (experimental system, device, machine, etc.);
- Abstract-logical modeling - models are presented by means of mathematics.

Modeling methods:

1) *Physical modeling* – models reproduce studied phenomenon in kind, while retaining its physical nature. For this purpose are used specially built physical models to which experiments have been conducted. The extent and nature of similarity between the model and the original is carried out by so-called criteria of similarity.

2) *Mathematical modeling* – models are built by means of mathematics and are separate equations or systems of equations (algebraic, differential, integral, etc.).

Mathematical models can be classified according to the following features [1, 2]:

- According to the type of restrictive conditions and the target function, mathematical models are *linear and nonlinear*. In common mathematical models are linear because finding an optimum program of linear models is considerably easier. For finding solutions of nonlinear models exist associates or iterative methods that lead to solutions close to optimal. However, in many cases, the results obtained from non-linear models significantly more realistically reflect the actual process reviewed.
- According to the preliminary nature of the information, models are divided into deterministic and stochastic. Deterministic models are those in which

with full accuracy are formulated both the restrictive conditions and the criteria for optimality. Stochastic models are those where there is uncertainty, i.e. the source data of the model are of probabilistic nature. They should be evaluated for the distribution laws of the respective random variables, which is not always possible. Deterministic models are much simpler than stochastic. So often instead of stochastic is considered deterministic model, ignoring the randomness of used variables.

- Mathematical models can be further divided into static and dynamic. The established mathematical model is static, when testing a process in a fixed period of time, without taking into account changes occurring in time. Dynamic models take into account the dynamics of the development process.

There are two groups of methods for constructing mathematical models - analytical and experimental.

- Through analytical methods, the model is built by the laws describing the processes in the target object. There are also a number of assumptions, whereby is taken into account the relative importance of the parameters defining the target object. Some odds and dependencies are obtained experimentally. The mathematical model is checked for adequacy by specially fitted experiments and using special numerical criteria.

- At the base of the experimental methods for mathematical modeling lies the so-called principle of "the black box". The object of the study is presented formally as a closed part of the environment - a black box, where unknown to the researcher processes occur. The object interacts with its environment through a number of input effects - factors and multiple output options - reactions defining its operation and condition. On the basis of experimental information for input factors and output parameters is obtained sufficiently complete information about the object without knowing its internal structure. Through this information is built a model linking the input factors and output parameters. Factors have different nature and are divided into manageable and unmanageable.

The stages of constructing mathematical models through analytical methods are shown in Figure 1, and through experimental methods are shown in Figure 2. The models built under this scheme are only valid for the conditions under which the experiments were built.

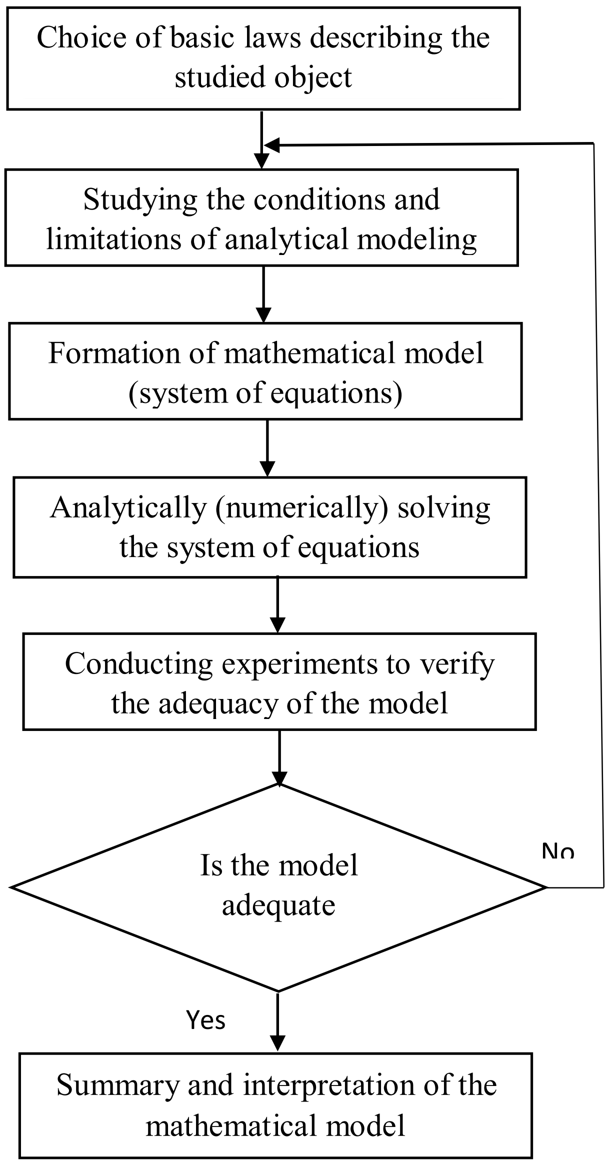

Figure 1: *Mathematical models by analytical methods*

A special case of mathematical modeling is *the simulation modeling*. Simulation modeling is a research method, in which the system of examination has been replaced by the model with sufficient accuracy. The aim is to experiment with the model in order to obtain

information about the real system. Experimenting with the model is called imitation.

The simulation model is a formalized description of the studied phenomenon in its entirety. Most often it is a common logical-mathematical representation of a system or operation, programmed to be solved by a computer system. One of the disadvantages of the simulation modeling is that the resulting decision always brings a private nature, responding to fixed values of the system parameters, input data and initial conditions. However, the simulation modeling is the most effective method for the study of complex systems (sometimes practically the only possible means of obtaining the necessary information for system behavior).

Another disadvantage is that the construction of the simulation model of a system is too complex and laborious process, also requiring high various qualifications.

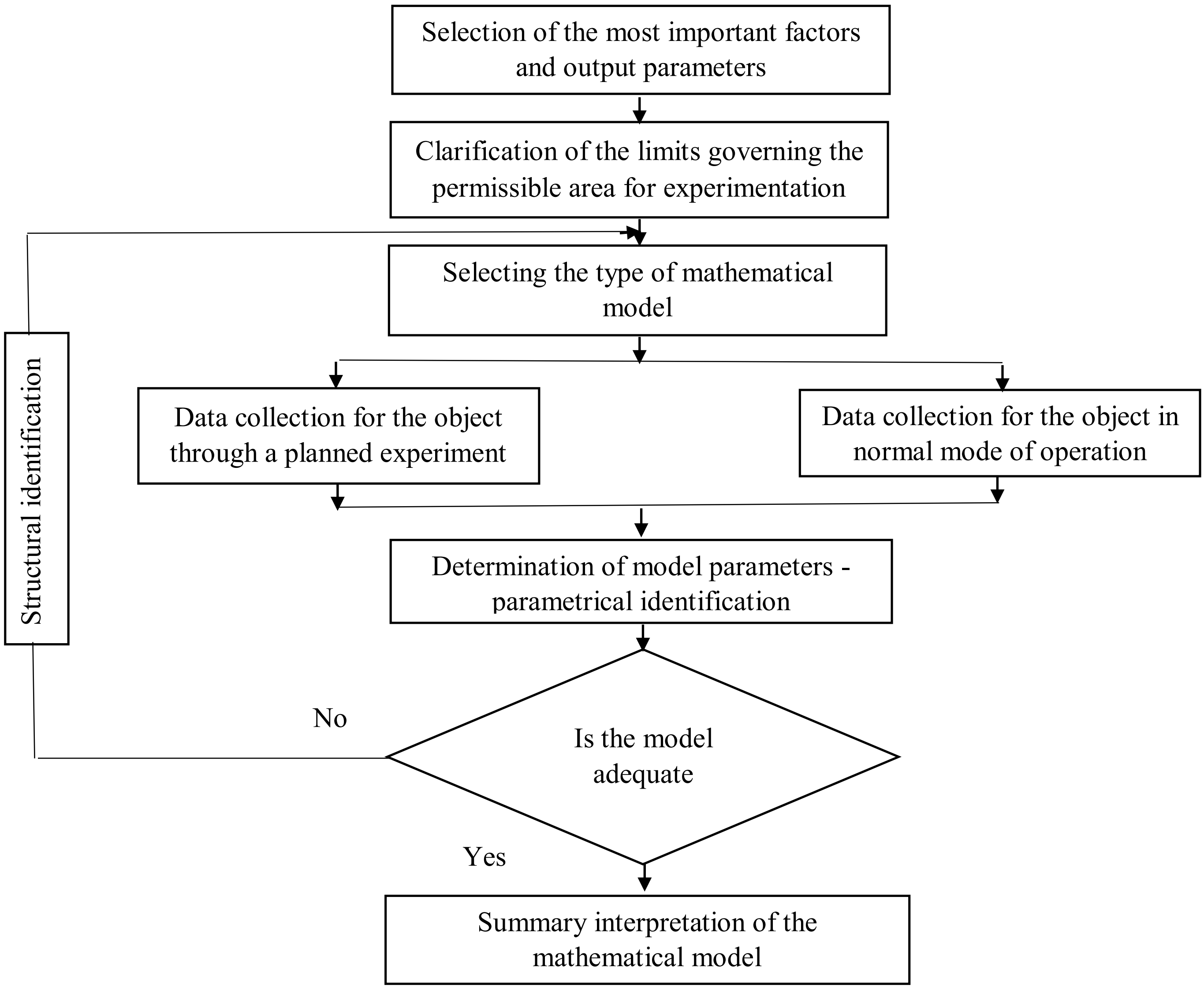

Figure 2: *Mathematical models based on experimental data*

In the social and natural fields, phenomena have mass character and therefore their knowledge and management can be analyzed only by the statistical approach. Regularities, detected by statistical study, are vital in the development and application of statistical and mathematical models of the examined processes. Studying past and present development of the phenomena and considering the fact that a sufficiently large number of observations establish lasting causalities and trends allows to predict the future development of the phenomena and with sufficient accuracy to provide the expected consequences of this from various controlling impacts. Thus open up new ideas and approaches to the management, and the most appropriate and effective management decisions can be made.

Statistical survey of relationships and dependencies in natural disasters

The study of relations and interactions between the phenomena and processes in natural disasters is one of the most important and complex tasks that stand before scientific knowledge. By analyzing the relationships and dependencies are revealed and learned the laws of the state and development of the phenomena and processes. The analysis of the causes of a condition or any subsequent changes, makes it possible to identify measures to regulate the management process. To take measures to eliminate confounding factors (errors), it is necessary to explore and analyze their impact on specific conditions. Disclosure of various factors and reasons justifying deviations from the normal rhythm of work are carried out by specially organized research. Through statistical analysis are examined the quantitative relationships between influencing factors and results.

The specific tasks in the study of relationships and dependencies are:

- To investigate whether the relationship of interest between the phenomena objectively exists and how it is manifested in practice. Here, when examining, methods to test the hypotheses are applied.
- To model existing dependencies. Regularities, to which is subordinate the dependence, is represented by a mathematical expression.
- To find the quantitative ratios, which are generated by the studied relationships and dependencies.

Statistical estimates and their properties [4, 5]

Let us assume that as a result of experiment is obtained the sample of test data $V = \{ v_1, v_2, \ldots, v_m \}$ the distribution of which has a known mathematical form but containing some unknown parameters. Basically there are infinitely many functions of the sample that can be offered as estimates of parameters of interest. Therefore arises the problem of selecting the most accurate assessment. It should be borne in mind that since each estimate is a function of the sample, it is an observed value of a random variable. Therefore cannot be predicted the individual importance of assessment in this particular case and the quality of the assessment should be judged only on the distribution of its meanings, which manifests itself in a long line of tests. Hence:

$$\hat{a}_j = R_j(V), \qquad j = 1,2,\dots \tag{1}$$

and here $\hat{a}_j$ is the assessment received by function R_j of the sample V, and "a " is the point whose values are the real searched meanings of the parameters.

For comparing the estimates obtained by different statistical methods are introduced the following main characteristics of the assessments [3, 4, 5, 6]:

• *Unbiasedness*. Let $M[\hat{a}_j]$ denote the mathematical expectation of the random variable $\hat{a}_j$. If $M[\hat{a}_j] = a$, the assessment $\hat{a}_j$ is called unbiased.

Unbiased estimates in some cases prove to be too complex functions of the results of measurements. Therefore, to simplify calculations we use estimates, where the absolute magnitude of the shift in 0 $m \to \infty$ tends to 0. Such assessments are called asymptotically unbiased.

Effectiveness. If the bulk density distribution of $\hat{a}_j$ is concentrated in a small neighborhood around the point a, then it is likely to be considered that the assessment differs from the true meaning only by a small quantity. In this sense, the assessment is the more accurate, as the less the dissipation of its true meaning is. Kramer has shown [5] that for some general conditions the average square deviation $M[\hat{a}_j - a]^2$ is limited from below by a positive number c depending only on the function of the distribution of $\hat{a}_j$, the size of the sample m and the displacement $b(a)$ which is given by the equality

$$M[\hat{a}_j] = a + b(a) \tag{2}$$

The ratio:

$$e[\hat{a}_j] = \frac{c}{D[\hat{a}_j]} \tag{3}$$

where $D[\hat{a}_j]$ is the dispersion of the random variable $\hat{a}_j$, is called the efficiency of the assessment $\hat{a}_j$. It is evident that:

$$0 \le e(\hat{a}_j) \le 1. \tag{4}$$

If in (4) is reached equality on the right, it means that $D[\hat{a}_j]$ has received its nominal possible value and $e(\hat{a}_j) = 1$. If estimating only one parameter, and the assessment by unbiased "c" is represented by the famous inequality Cramer-Ra [3, 4, 5]. In [6] it is shown that the above considerations are valid and when the evaluated parameters are more than one, but not infinite. However, the generalized dispersion is used, which is the determinant of the covariance matrix of the multi-dimensional random variable $\hat{a}_j$.

The term "effective assessment" was introduced by Fischer and as noted above expressed strictly mathematical our intuitive understanding of the most accurate assessment. Kramer showed [6] that the effective assessment of a single parameter is the only one valid.

If in effect:

$$\lim_{m\to\infty} e(\hat{a}_j) = 1 \tag{5}$$

then this assessment $\hat{a}_j$ is called asymptotically effective.

•*Consistency***.** The assessment $\hat{a}_j$ is called consistent if during an unlimited increase in the volume m of the sample V, it strives more likely to "a", i.e.:

$$\lim_{m\to\infty} P(\{\hat{a}_j - a\} < E) = 1 \tag{6}$$

where P is the probability of the event $\{\hat{a}_j - a\} < E$, and $\{x\}$ is the norm of the vector x, which is defined by the terms:

1) $\{x_1\} = 0$, if $x_1 = 0$, and $\{0\} = 0$;

2) $\{cx_1\} = |c|\{x_1\}$, in any numerical factor "c";

3) $\{x_1 + x_2\} \leq \{x_1\} + \{x_2\}$.

The choice of one or another rate is determined by the terms of the statistical task that must be resolved.

•*Sufficiency***.** Let us assume that the family F of admissible values of the sample V is set. Statistics s is a function of observations $s = S(V)$ and in the general case S is a vector.

Statistics s [4, 5] is called sufficient for the family F if the conditional density of distribution of V to s remains the same for all distributions of F.

The term ''sufficient statistic'' was introduced by Fischer to denote these statistics, which contain all information about estimated parameters that can be derived from the available sample.

•*Minimum sufficiency.* Statistics s is called minimum sufficient [4, 5], if while storing the property sufficiency it is impossible to further reduce the data compared to s.

As far as $\hat{a}_j$ is a random variable, then it is not possible to accurately determine the degree of proximity of $\hat{a}_j$ to a, but if you know the laws of distribution of the considered parameters it can be found how likely it is that these parameters turn out to be in a series of measurements within certain limits. Such an approach to the evaluation of search parameters is called evaluation using confidence intervals. Under confidence interval we mean one with random edges that with sufficiently high probability covers the true meaning of the unknown parameters.

From the considerations up to here it appears that the choice of methods for statistical processing of the results of research in order to achieve high accuracy it is necessary to use:

1) minimal sufficient statistics, which allows by using minimum statistical material to find the assessments in search;

2) constant, effective and cogent assessments.

Conclusion

Regularities which are detected by statistical study are vital in the development and application of statistical and mathematical models of disastrous processes. Studying past and present development of the phenomena and considering the fact that a sufficiently large number of observations establish lasting causalities and trends allows to predict the future development of the phenomena and with sufficient accuracy to foresee the expected consequences of any type of controlling impacts. Thus new ideas and approaches to the management open up and the most appropriate and effective management decisions can be taken.

To study the relationships and dependencies various methods are used. The choice of a method depends on the purpose of the survey and the available information. There are various possibilities for combinations of the signs by which to be presented a given dependency to reality. According to these combinations are used different methods of examination of the considered relationships.

Acknowledgments:

This paper is supported by the Project BG051PO001 - 3.3.06 - 0003 “Building and steady development of PhD students, post-PhD and young scientists in the areas of the natural, technical and mathematical sciences”. The Project is realized by the financial support of the Operative Program “Development of the human resources” of the European social fund of the European Union.

REFERENCES:

[1]. Atanasov B., Quantitative Methods in Business Management, kn.1, Varna 1994
[2]. Dimanova D., "Theory of modeling", Association Scientific and Applied Research, International Journal, Vol. 4, 2013, ISSN 1314-6289
[3]. Dimitrov B, Janev N., Probability and Statistics, Sofia 1998
[4]. Gatev K., General Theory of Statistics, Sofia 1983
[5]. Gmurman V. E., Probability Theory and Mathematical Statistics, Moscow, 1977
[6]. Rumshiskij L. Z., Mathematical processing of experiment results, Moscow, 1971