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TRANSPORTATION SYSTEMS MODELLING

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Abstract: *As it is with every activity, the transport network can be organized in many ways by using different technical means and technologies. Taking certain management decisions is connected with the choice of one of the many possible options for realization of the transportation process. This is done with the help of one or a few indicators which serve as a criterion of efficiency, e.g. exploitation costs, profit, safety, time, etc. The research of the transport objects functioning and the optimization of their parameters is a very complicated task in most of the cases. In such cases the solutions of the problems which concern us are derived by the research and experimentation with a substitute (analogue) of the real object, specially designed for this purpose. This analogue is called a model and the process of its creation is called modelling.*

Key words: *modelled object, efficiency criteria, optimization of parameters*

Methodology of creation and evaluation of the models

The development of a model includes the following stages:

• conception of the problem:

– to clarify the definition of optimality (rationality), i.e. which parameters are going to be used as criteria for its functioning (efficiency criteria);

– to clarify the range of the problem and the size of the tasks, i.e. the possibility of the system to be decomposed. The decomposition is necessary in order to simplify the models and reduce them to such for which there are efficient algorithms for their solutions. [5]

– to define the input controllable parameters (optimizable, which can be changed by the managing authorities) and uncontrollable parameters (the ones whose values influence the optimality criteria but for one reason or another, they are not subject to purposeful change by the authorities which control the object).

• building the model – finding the correlation between the controllable and uncontrollable parameters on the one side and the efficiency indicator on the other side.

As a final result, the development of the model is resolved by finding the correlations:

$$z_i = f_i\left(x_1, x_2, \dots, x_j, \dots, x_n\ ;\ c_1, c_2, \dots, c_k, \dots, c_n\right) \quad (1)$$

where: z_i are the chosen adequacy criteria;

x_j – the controllable parameters;

c_k – the uncontrollable parameters.

When developing the model, different kinds of ratios are used:

- ratios coming from definitions, for example

  profit = income – expenses;
- empirical ratios – they are being used when it is difficult or even impossible to directly define the existing connections in the model;

• collection and processing of the necessary information – when gathering and processing of the collected data, the model sensitivity towards the unknown parameters should be considered. If the modelled object exists, the data is gathered directly about it. If it does not exist, we use data from an analogue object or from references.

• model check – an obligatory stage in the solution of a problem.

Specifics of the transportation models

The transportation systems are complex and dynamic. They consist of two types of basic elements:

• Rolling stock – wagons, automobiles, etc;

• Fixed facilities – roads, ports, train stations, safety and management tools, etc.

The fixed facilities influence directly the operation of the rolling stock and together they define the transportation capability of the transportation object and to a certain degree – the quality characteristics of the transportation process (speed, frequency, safety, etc.)

The vehicular process makes the cohesion between the transportation system and the customers (shippers). The main goal of the general transportation models is to show the relationships between the parameters (technical and technological of the fixed facilities and the rolling stock and the customers' requests. In general, these relationships are shown on Figure 1.

Due to the great complexity of the transportation system, it is necessary to separate a certain object or problem, i.e. the system (model) is decomposed. The basic condition which gives possibility to decompose the model is the relative independence of the separate submodels. This is why it is important for the optimum values of the included parameters to be weakly dependent on the condition and operation of the other systems.

Due to the presence of two-sided connections, presented on Figure 1, the optimization of the transportation models is done iteratively. Firstly, an internal optimization of the decomposed objects (models) is performed and then they are considered as uncontrollable. The more complicated objects are looked into successively until the given problem is solved. After that we start all over again as we decide how much the end solutions influence the parameters of the decomposed models. This continues until convergence is achieved, i.e. until minor changes in the final results.

The transportation process depends on many external factors, and the basic ones are: the customers' behavior and the natural resources. Within the limits of the transportation system, they are uncontrollable and consequently they have very distinctive characteristics. In this way the transportation systems have probabilistic (stochastic) character. In order to model such objects, contemporary methods for transportation systems modeling are being used.

The process in which all transportation operations and the customers' requests happen at certain moments (according to a schedule) is called determined. In order to describe the determined systems, we use methods like schedules, network planning, schedule theory, etc. but their application is comparatively limited because it is not possible to make models which include all the elements of the transportation process with their help.

Such models can be created with the help of Theory of Mass Service (TMS)

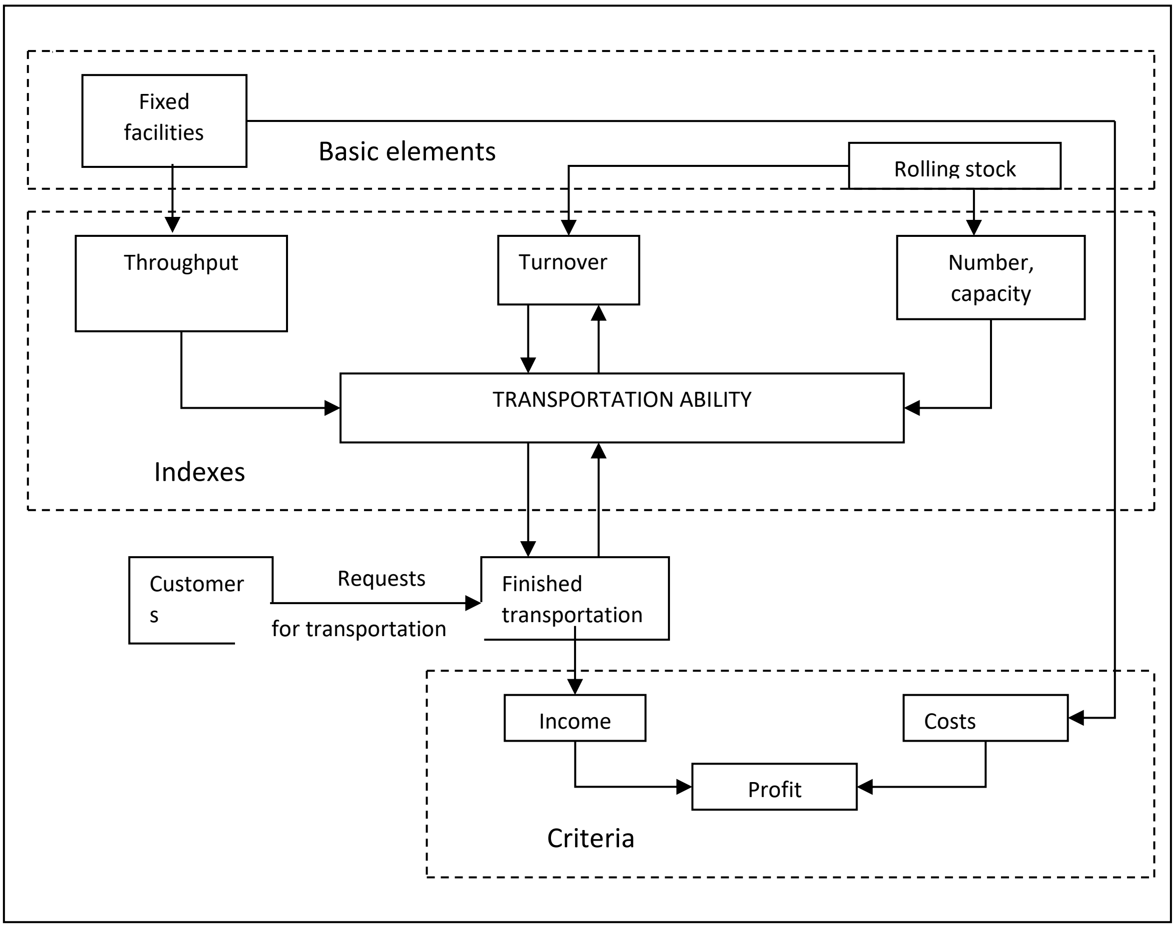

Fig.1. Relations between the parameters of the fixed facilities, the rolling and the customers

Modeling of transportation systems with the help of the Theory of mass service (Queuing theory)

The processes going on in the transportation are complex and various. Regardless of the existing differences between them, due to the type and specialization of the performed transportation operations, the common connection between the transportation processes is their customer services activities.

The basic task of the transportation is to satisfy its customers' needs. The whole transportation process consists of a series of services. The customers are served by the rolling stock, the rolling stock – by the fixed facilities, etc. and this is shown on Figure 1.

The Theory of mass service is created by the needs of the practice to analyze processes which lead to accumulation, keeping and creation of queues when servicing. By using TMS we study the processes of constant occurrence and satisfaction of the requests when performing certain tasks. The requests occur, they are generated by the serviced system and they are received and processed by the servicing system. The aggregate of the two systems is called system for mass servicing, in which the flow of requests, the line (queue) and the channels for services are consistently linked.

The so-called Systems of mass service (SMS) are object of the Theory of mass service. The structure of the SMS is defined by its contents and its functional links. It consists of input of requests, channels of service (machines, facilities, etc.) and output of processed requests, shown on Figure 2.

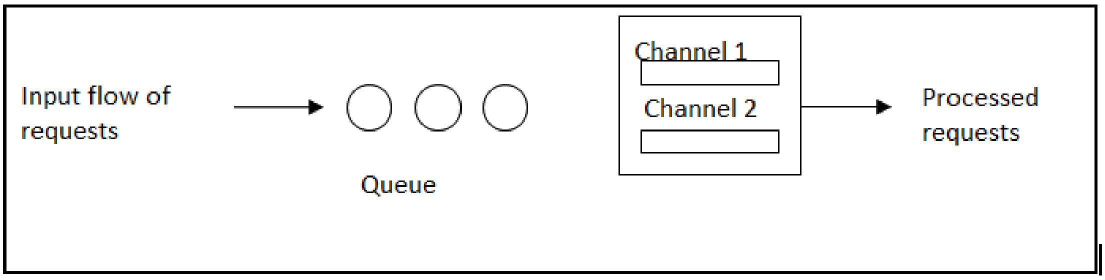

Fig.2. Structure of the System of mass service

In the SMS a certain flow of requests is being serviced by one or more than one Servicing equipment (SE) and while being served, some requests receive rejections or downtime in the queue.

It is the purpose of TMS to define the indexes of work of SMS. One SMS is defined by the input flow, service mechanism and queue discipline.

The flow of requests is characterized by intensity of the input flow λ (requests for a unit of time) and by the law of distribution of time between receiving the requests.

The service is defined by the number of servicing equipment S, the servicing intensity μ and the law of distribution of the time for servicing one request $t_{об}$.

The following relationship is valid

(2) $$\mu = \frac{1}{\bar{t}_{об}}$$

where $\bar{t}_{об}$ is the average time for servicing one request.

Queue discipline refers to the rules, in accordance to which the servicing mechanism receives the incoming requests as: first come – first served; last come – first served or random choice of requests. The last rule is used often in the transportation systems because it allows operational impact upon the system.

We should refer “priority” servicing, as well as the individual behavior of the customers (refusal of service, migration to other queues, etc.) to queue discipline.

The systems for mass servicing are characterized by load ρ

(3) $$\rho = \frac{\lambda}{\mu}$$

The compulsory requirement for the work of the systems for mass servicing without rejections is $\frac{\rho}{s} < 1$. If this requirement is not kept, the number of the requests in the que increase indefinitely. For single channel SMS $s = 1$ and the requirement is $\rho < 1$.

The indexes for work of SMS are divided into *averaged*, defined by mathematical expectation of the given value, and *probabilistic*. The most important of them are:

- average numbers of requests in the system L_s;
- average number of requests in the queue L_q;
- the probability the system to have *i* requests P_i;
- average time for staying of the requests in the system T_s;
- average time for staying of the requests in the queue T_q;
- rejection probability $P_{\text{от}}$ in the systems with rejection;
- the probability of a request to wait P_w;
- probability all channels to be busy P_d.

Classification of SMS

According to the number of simultaneously working servicing equipment:

- single-channel;
- multiple-channel.

According to the formation of the queue:

- with rejections;
- with limited queue;
- with limitless queue.

According to the discipline of the service:

- priority service;
- random service.

According to the number of successive servicing equipment:

- single-phase;

- multiple-phase.

According to the source of requests:

- closed systems (limited source);

- open systems (limitless source).

The laws for the distribution of the service times and the input flow are presented with the following contractions:

$\boldsymbol{M}$ – Poisson input flow (exponential distribution of the service time);

$\boldsymbol{D}$ – determined input flow (constant service time);

$\boldsymbol{E_k}$ – Erlang flow and distribution of the service time from k-order;

$\boldsymbol{G}$ – random function of distribution.

In transportation the systems with limitless queue are more common. For brevity the SMS are written with the above-mentioned symbols for the distributions of the input flow and the service time and after that we put *1* when the system is single-channel and *S*, when it has *S* servicing devices. For example, E_k/S means multi-channel SMS with Poisson input flow and Erlang distribution of service time from k-order. The following correlations are valid for all the SMS:

(4) $T_s = \frac{L_s}{\lambda}$, average time in the system

$L_s = E(l_s) = \sum_{i=1}^{\infty} iP_i$ average number of requests in the system

(5) $T_q = \frac{L_q}{\lambda}$, time / request

$L_q = E(L_q) = \sum_{i=1}^{\infty}(i - S)P_i \quad at\ i > \rho$ requests

(6) $L_s = L_q + \bar{S}$

Where $\bar{S}$ is the average number occupied channels and l_s and l_q are respectively number of requests in the system and the queue in *i* moment in time

At SMS without rejections

(7) $\bar{S} = \rho$

(8) $T_s = T_q + \bar{t}_{об}$

The shown correlations are known as Little's formulas. They allow the definition of every one of the given values if one of them is known. (Figure 3).

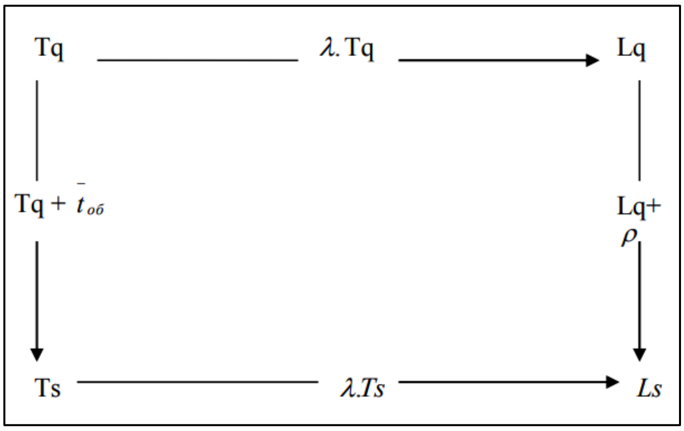

Fig.3

Modeling of multiple-channel closed SMS (end source)

In the closed systems the requests sources are limited and fixed. Service requests come from every source and at random time interval t. The intensity of the request's reception from one source λ depends on the average time $\bar{t}$ of the intervals from the coming out of the request until its reception, shown on Figure 4 and the following formula is valid:

(9) $$\lambda = \frac{1}{\bar{t}}$$, requests / time

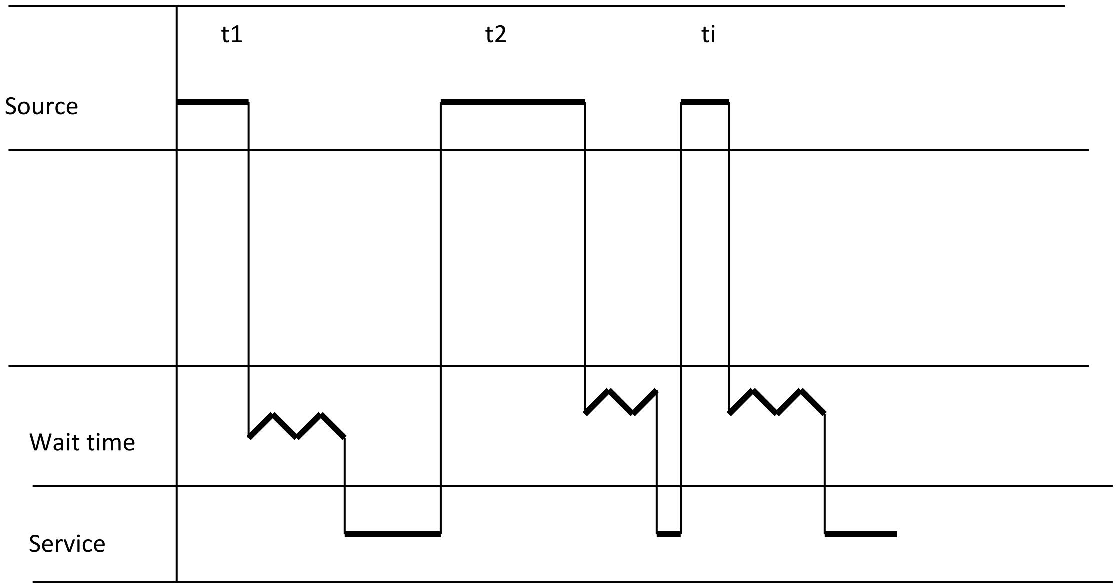

Fig.4

The sum intensity of the input flow λ_p is a variable which depends on the requests number in the SMS:

(10) $$\lambda_P = (N-1)\lambda \text{, requests / time}$$

The closed systems are harder to describe. There are formulas only for SMS with Poisson input flow and exponential service time. Poisson flow refers to the flow, coming out of one source, i.e. the times *t* in Figure 4 and distributed according to an exponential law.

Poisson law. Poisson law is closely connected with the term *Poisson flow*. If at the occurrence of certain homogenous events in time the following conditions are met: *stationarity, ordinariness, and lack of consequences,* the number of the events for a unit of time is a random value, distributed according to the Poisson law.

Stationarity means that the average number of events does not change in time.

Ordinariness means that the probability for the simultaneous occurrence of two events equals zero.

The absence of consequences expresses the requirement of independence between the number of occurred events before and after a given random moment. That means that there is no connection or there is a very weak connection between the moments of the events' occurrence. That condition is often violated which leads to deviations in the Poisson character of the flow.

Exponential law. It is an analogue of the Poisson law at continuous random values. It has been proven that if one flow is Poission flow, the intervals between the reception of the requests are distributed according to the Exponential law. We can expect that according to the Exponential law the following are distributed: the intervals of the coming of the means of transportation, the times of faultless work and recovery, some of the times for service, etc.

The real transportation systems are formally closed. But these systems are referred as systems with endless source and limitless queue. This assumption is due to the comparative equality of the parameters of the two systems at a big number of sources. The result is *M/M/S* with *N* sources of requests.

(11) $$P_0 = \left[\sum_{i=0}^{S} \frac{N!\rho^i}{t!(N-i)!} + \sum_{i=S+1}^{N} \frac{N!\rho^i}{S^{i-S}S!(N-i)!}\right]^{-1}$$

(12) $$P_i = \frac{N!\rho^1}{t!(N-t)!}P_0\ , \quad i < S$$

(13) $$P_i = \frac{N!\rho^1}{S^{i-s}S!(N-t)!}P_0\ , S < i \leq N$$

where $\rho = \frac{\lambda}{\mu}$ cannot be defined as an overload of the system. Because λ is an entry only from one source. The load of the system ρ_p is defined like an average value.

(14) $$\rho_p = E\left(\frac{\lambda_p}{\mu}\right) = \frac{\lambda}{\mu}\sum_{i=1}^{N}(N-i)P_i = \left(\sum_{i=1}^{N} NP_i - \sum_{i-1}^{N} iP_i\right)$$

Because

(15) $\sum iP_1 = L_s$ то $\rho_p = \rho(N - L_s)$

The average size of the queue is derived from:

(16) $$L_q = \sum_{i=S}^{N}(i-S)P_i$$

The average time of staying in the system T_s and in the queue T_q can be defined by using Little's formulas (5) and (6) – the intensity of the total input flow λ_p is defined by $\lambda_p = \rho_p\mu$.

Using the Multichannel closed SMS (end source)

In a distribution point two loading-unloading machines work with equal efficiency. Six automobiles are attached to the distribution point and they transport load from and to a warehouse. The time for one cycle of a car without the time it stays in the distribution point is distributed according to the Exponential law with an average value 1 hour. The automobiles are processed average 15 minutes by any of the machines. The processing time is distributed according to the Exponential law. If an automobile costs 5 BGN, how much is the daily loss from the downtime of the automobiles which are waiting to be processed for two shifts, each 8,5 hours long.

Solution – because the number of the automobiles is limited to 6, the system is closed (with an end source). The automobiles are processed by any of the two machines, so the system is two-channel. There is no data for priority service. The result is *M/M/2* for 6 requests.

λ= 1 requests from one automobile / hour

$\mu = \frac{60}{15} = 4$ requests/hour, $\rho = \frac{1}{4} = 0.25$

The probabilities of the conditions are defined by (11), (12) and (13)

$$P_0 = \left[\sum_{i-0}^{2} \frac{6!\,0.25^i}{t!\,(6-i)!} + \sum_{i=3}^{6} \left(\frac{6!\,0.25^i}{2^{i-2}2!\,(6-i)!}\right)\right]^{-1}$$
$$= (1 + 1.5 + 0.94 + 0.47 + 0.17 + 0.042 + 0.005)^{-1} = 4.127^{-1}$$
$$= 0.242$$

$P_1 = 0.363, P_2 =$0.228, $P_3 = 0.114$, $P_4 = 0.041$, $P_5 = 0.01$, $P_6 = 0.0012$

From formula (16) we get:

$L_q = \sum_{i=3}^{6}(i-2)P_i = 1 \cdot\ 0.114 + 2 \cdot 0.041 + 3 \cdot 0.01 + 4 \cdot 0.0012 =$ 0.23 automobiles.

To find T_q it is necessary to know the sum intensity of the input flow $\lambda_p = \rho_p\mu$. From (12) $\rho_p = 0.25(2 - L_s)$

$L_s = \sum_{i=1}^{6} iP_i = 1 \cdot 0.363 + 2 \cdot 0.228 + \cdots. +6 \cdot 0.0012 = 1.38$ automobiles

$\rho_p = 0.25(2 - 1.38) = 1.155,\qquad \lambda_p = 1.155 \cdot 4 = 4.62$

$$T_q = \frac{L_q}{\lambda_p} = \frac{0.23}{4.62} = 0.0498 \text{ hours/automobile}$$

For 24 hours the total work time is T = 2· 8.5 = 17 hours. During this period $T\lambda_p = 17 \cdot 4.62 = 78.54$ automobiles have passed through the system.

The value should not be rounded to the nearest whole number because it means the average number of automobiles per shift. Every automobile stayed in queue an average T_q hours. Then the total automobile-hours B for 24 hours are $B = T\lambda_p T_q = 78.54 \cdot 0.0498 = 3.91$ automobiles per day.

The total loss E would be $E = 5B = 19.55$ BGN/day.

The total automobile-hours can be calculated if we consider that L_q is the average number of waiting automobiles. Then $B = T \cdot L_q = 17 \cdot 0.23 = 3.91$ aut.-h/ day.

The new information is used to make management decisions and if they lead to improvement of the object functioning, we can consider that the model has been created successfully. Consequently, the adequacy can be defined as the quality of the model to give enough information which allows us to make correct management decisions.

In order to model the complicated transport systems, it is suitable to look at them as a network of intrinsically linked and interacting technological systems. The processes in these systems can be modelled with the help of the Theory for mass service (TMS). In order to do so, they have to be presented as Systems for mass service (SMS) and we have to consider the influence of random factors upon the intervals of time between the receiving of the requests of the SMS as well as upon the time for service of the requests in the different SMS.

Using TMS for modelling the real processes is connected with knowledge about the laws for distribution of the input flow of requests and the times for their service.

Sometimes the complicated processes in the transport cannot be modelled satisfactorily with the help of the analytical models of the Theory of mass service. When we lack analytical end formulas for part or for all of the operational characteristics, we can use the method *imitational modeling*, also known as the Monte Carlo method, which presents multiple realizations of the modelled process and considering the values which are sought by the researcher.

Basic problems, solved with the help of TMS

We can solve a broad spectrum of problems with the help of TMS. Depending on their characteristics, we choose the criteria evaluation and the necessary preciseness of the input data.

- Choice of appropriate organization

It is supposed that the different versions of the organization are not connected with substantial expenses. This is the reason why they are compared according to natural indicators and the most convenient of them is the time for stay in the system T_s. The criteria for preciseness when defining λ, μ and ν are not very demanding. Often the problems can be solved theoretically.

- Choice of optimal technical equipment

The improvement of the technical equipment of the SMS elements is always connected with additional costs but it decreases the losses from the downtime of the requests in the system E_n, the customers' quitting E_k, etc. For a certain period of time T, the requests that are passing through the system are $\lambda \cdot T$.

(17) Then $E_n = \lambda \cdot T \cdot T_s \cdot c_n$, BGN

where c_n are the expenses for a unit of time when the request is in downtime.

(18) $E_k = \lambda \cdot T \cdot P_{\text{от}} \cdot c_n$, BGN

where $P_{\text{от}}$ is the possibility the customer to quit and c_k are the losses from one abandoned request.

The possible technical solutions are increase of the number of servicing equipment or increase of the service intensity.

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