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COMPARATIVE ANALYSIS OF THE PROBABILITY IN DIGITAL WIRELESS COMMUNICATION SYSTEMS

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ABSTRACT: Digital signals dominate satellite communications systems. A measure the quality of digital signals is Bit Error Rate (BER) and Symbol Error Rate (SER). The main purpose of the paper is a comparison of the probability of error for various modulation and demodulation schemes used in digital communications systems (DCS) for transmitting information over channel. Error Rate parameters are resulting analytically, not experimentally.

KEYWORDS: the probability of error, modulation and demodulation schemes, wireless communication systems.

The purpose of Channel Coding is decreasing of errors when signals are transmitted by noisy channel. The capabilities of Channel Coding were investigated by Claude Elwood Shannon (30.04.1916–24.02.2001) and written in “A mathematical theory of communication” [3].

A measure the quality of digital signals is Bit Error Rate (BER). The bit error rate or bit error ratio is the number of bit errors divided by the total number of transferred bits during a studied time interval. BER is a unitless performance measure, often expressed as a percentage number. As the custom is the errors are slightingly small if the BER values are 10^{-5} for the audio signals and 10^{-11} for the data. In cases when the states of signal information parameter are represented by number of bits (the binary bit stream is combined into groups, called symbols) there is in use Symbol Error Rate (SER). The parameters P_b (BER) and P_S (SER) can be found by simulation, real experiment or analytically.

The BER is often expressed as a function of the E_b/N_0, (energy per bit to noise power spectral density ratio):

(1) $$BER = \frac{1}{2} erfc(\sqrt{E_b/N_0}),$$

where $erfc$ is a Complementary Error Function.

For transmission the digital information signal is gone to the modulator to form a radiosignal. The carrying of signal spectrum from low frequency to high frequency is doing by modulation. Modulation is a process of transfer of information over sinusoidal microwave carrier. Thus, in transmitter enter two signals - information low frequency baseband signal $s_m(t)$ and continuous in time high frequency signal $s_c(t) = f(t; a_1, a_2, ..., a_m)$. The combination of parameters $\{a_i\}$ determines a signal form. When the value of some of these parameters depend on the information signal $s_m(t)$ in time or other variable quantity, the form of high frequency signal is carrying an information that is identical with this in $s_m(t)$. Then the signal $s_c(t) = f(t; a_1, a_2, ..., a_m)$ is called Carrier. The low frequency signal $s_m(t)$ is called Modulating Signal, the result of modulation - Modulated Signal. The general form of the carrier wave is harmonic:

(2)

$$s_c(t) = A_i.\cos(2\pi.f_0.t + \varphi) = A_i.\cos(\omega_0.t + \varphi),$$

where A_i is an amplitude of carrier, $\omega_0 = 2\pi f_0$ is a radian frequency in rad/s, φ is the phase.

The digital modulation is a process whereby the amplitude, frequency, phase or combination of them is varied in accordance with the information. In this reason, the main digital modulation method are called Amplitude Shift Keying (ASK), Frequency Shift Keying (FSK), Phase Shift Keying (PSK) and hybrid methods that are combination of them. The frequency and phase modulations are mutually connected and they are called Angle Modulations. When both the amplitude and phase simultaneously are changed the modulation is called Quadrature Modulation.

The receiver process the modulated signal by transforming in low frequency signal (demodulation) and then make detection. The demodulation is used to recovery waveform, and detection is the process of symbol decision. There are known two types of detection: coherent and noncoherent detection. Coherent detection is a process when the receiver utilizes information about phase of RF carrier. When the receiver doesn't use information about phase of RF carrier it is called noncoherent detection, the phase estimation is not required. When the signal is transmitted there are two main causes for error performance degradation. The first of them is intersymbol interference (ISI) because of nonideal system transfer function. The second is electrical noise and interference prodused by galaxy and atmospheric noise, intermodulation noise, switching transients and other sources. The additive white Gaussian noise (AWGN) is the most often used as model of the noise in the detection process. The detector has to retrieve the digital signal from the received waveform without of errors and maximum value of the ratio E_b/N_0, if it is possible.

The transmitted signal in the limits of symbol interval $(0,T)$ is

(3)

$$s_i(t)=\begin{cases} s_1(t), where\ 0\le t\le T\ \ for\ a\, binary\ 1;\\ s_0(t), where\ 0\le t\le T\ \ for\ a\, binary\ 0.\end{cases}$$

The received signal $r(t)$ is degraded by noise $n(t)$ and can be represented by

(4) $r(t)=s_i(t)+n(t),\ i=0{,}1;\ 0\le t\le T$,

where $n(t)$ represent a noise amplitude and it is changing from sample to sample.

Figure 1 shows the binary signal acted by random noise. The amplitude of received signal varies at each sample.

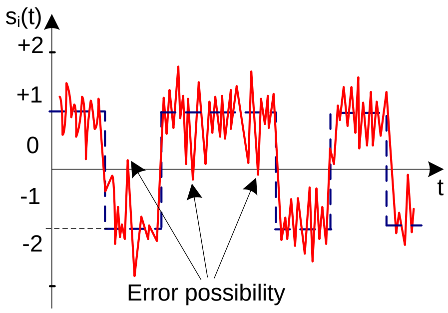


Figure 1. The binary signal acted by noise

The value of noise varies with time and it is assumed to be normally distributed with a zero mean and variance of quantity σ^2, where σ is a standard deviation of the noise process. The receiver has to make a decision on the fluctuating values, shown in Fig. 1. There are two possibilities to make a mistake: first of them is when signal $s_1(t)$ has been received, but because of the presence of noise the energy of the signal is not enough and the detector chooses the hypothesis H_2: $z(T)<\gamma$, as it was received the signal $s_0(t)$. The quantity $z(T)$ is the sampler output at the end of each symbol duration, and it has a voltage value directly proportional to the energy of the received symbol and inversely proportional to the noise [1].

(5) $z(T)=a_i(T)+n_0(T),\ i=0{,}1$,

where $a_i(T)$ is the desired signal component, $n_0(T)$ is the noise component.

The two error possibilities can be expressed as

(6) $$p(e|s_1)=\int_{-\infty}^{\gamma_0} p(z|s_1)dz\ ,$$

(7) $$p(e|s_0)=\int_{\gamma_0}^{\infty} p(z|s_0)dz\ ,$$

where

$$p(z|s_1)=\frac{1}{\sigma_0\sqrt{2\pi}}.\exp\left[-\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right]$$ and

$$p(z|s_0)=\frac{1}{\sigma_0\sqrt{2\pi}}.\exp\left[-\frac{1}{2}\left(\frac{z-a_0}{\sigma_0}\right)^2\right]$$ are

the conditional probability density functions, shown in Figure 2.

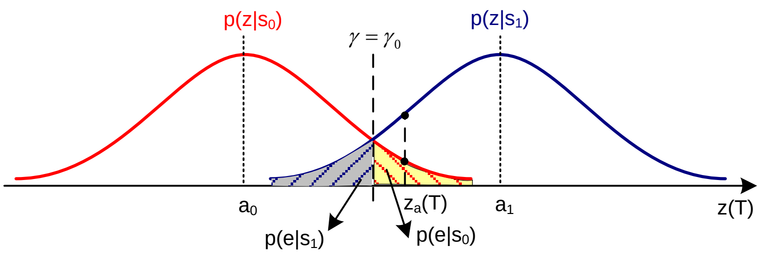


Figure 2. Conditional probability density functions

The function $p(z|s_1)$ shows the probability density of the random variable $z(T)$ if symbol s_1 was transmitted. Respectively, the function $p(z|s_0)$ shows the probability density of the random variable $z(T)$ if symbol s_0 was transmitted. The intervals are up to $\gamma_0 = \frac{a_1 + a_0}{2}$ symmetrically because of equal probability to receive 0 or 1 [10]. The Figure 2 illustrates the process of making decision. Choosing $z(T) > \gamma$ is equal decision that signal $s_1(t)$ was sent and a binary 1 was detected by the receiver. Respectively, choosing $z(T) < \gamma$ notes that signal $s_0(t)$ was sent and a binary 0 was detected.

Because of the symmetry of the probability density functions, and the equal of the priori probabilities the probability of bit error is

(8) $P_B = p(e \mid s_1) = p(e \mid s_0)$.

Therefore, the probability of bit error can be calculate by integrating of conditional probability $p(e \mid s_1)$ or $p(e \mid s_0)$ between their limits.

(9) $$P_B = \int_{\frac{a_1+a_2}{2}}^{\infty} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right]dz = \int_{u=\frac{a_1+a_2}{2\sigma_0}}^{u=\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{u^2}{2}\right]du = Q\left(\frac{a_1-a_2}{2\sigma_0}\right),$$

where $Q(x)$ is a tabular function, Gaussian error integral $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{u^2}{2}\right)du$.

The purpose of each detector is to minimise the probability of error. Thus the receiving filter has to be optimized by choosing the maximum argument of Q-function in Equation 9. It is known, that matched filter provides the maximum signal-to noise power ratio at its output at time $t = T$

(10) $$\max\left(\frac{S}{N}\right)_T = \frac{(a_1 - a_2)^2}{\sigma_0^2} = \frac{2E}{N_0},$$

where E is a energy of the input signal, N_0 is a noise power spectral density.

Thus, by maximizing the signal-to noise power ratio at the output, matched filter achieves a maximum distance between two possible signals $s_1(t)$ or $s_0(t)$, and provides better performance to detect signal without errors.

There are work out an equations, that show the probability of error for the different types of modulation.

- Binary Phase Shift Keying and coherent detection:

(11) $$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right);$$

- Differential Binary Phase Shift Keying and coherent detection:

(12) $$P_B = \frac{1}{2}\exp\left\{-\frac{E_b}{N_0}\right\};$$

- Orthogonal Frequency Shift Keying and coherent detection:

(13) $$P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right);$$

- Orthogonal Frequency Shift Keying and noncoherent detection:

(13) $$P_B = \frac{1}{2}\exp\left\{-\frac{E_b}{2N_0}\right\}.$$

Figure 3a shows curves for various binary modulations.

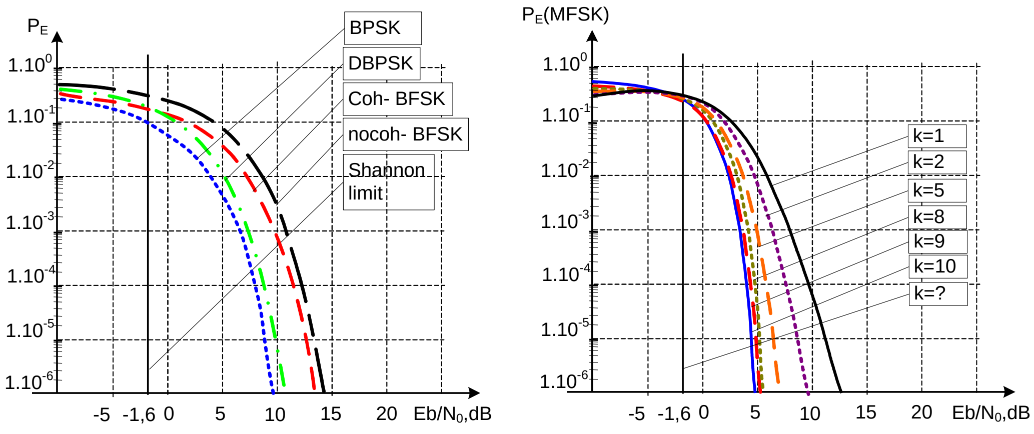
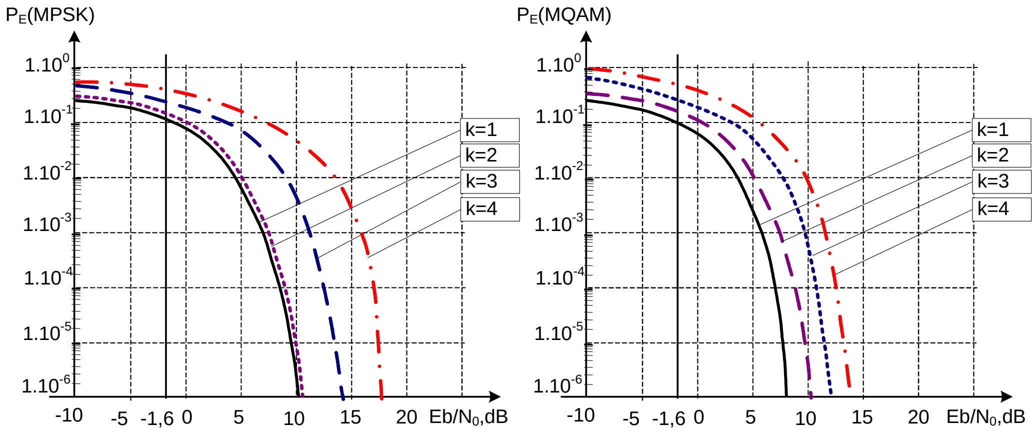


Figure 3. Curves for various binary modulations (a) and M-ary frequency shift keying (b).

The less value of E_b/N_0 ratio achieves BPSK modulation, the maximum value - nocoh-BFSK with the equal probability of error. The difference between them is about $4dB$. The DBPSK lose $1dB$ from BPSK because of character - it redoubles incoming errors. Although DBPSK is less efficient than BPSK, sometimes it is preferred modulation because DBPSK receiver doesn't need phase synchronization. BPSK works with antipodal signals. The distance between antipodal signals is $2\sqrt{E_b}$. The distance between orthogonal signals (FSK) is less $\sqrt{2E_b}$, i.e. the distance squared between orthogonal signals is two less then distance squared between antipodal signals. This is the reason of higher probability of error when FSK is detected. The comparison of coherent OFSK with noncoherent OFSK shows that noncoherent OFSK requires approximately $1dB$ more E_b/N_0 ratio. It is because the error performance of this type of modulation depends on bandpass filter bandwidth. The probability of error P_b becomes more when bandwidth increases [1]. The minimum bandwidth allowed is equal to bit rate.

Figure 3b shows curves of error probability depending on E_b/N_0 ratio for coherent M-ary Frequency Shift Keying and different symbol length. Figure 4a shows curves of error probability depending on E_b/N_0 ratio for M-ary Phase Shift Keying, Figure 4b - M-ary QAM.



a) b)

Figure 4. Curves for various M-ary modulations and different symbol length.

Figure 3b shows as number of bits *k* in symbol increases the error probability decreases. The required bandwidth also increases. For the M-ary phase curves in Fig. 4a as *k* increases the required bandwidth decreases if data rate is fixed, or if the bandwidth is fixed as k increases the data rate also increases. In this reason more length of symbol leads to less error performance. Therefore, M-ary modulations can be utilize to achieve optimum in accordance to required performance of the link - more data rate in the same bandwidth and less error performance, or less error probability versus data rate and bandwidth.

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