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INTERPOLATION OF RASTER SURFACES (GRID INTERPOLATION).THE KRIGING METHOD

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ABSTRACT: *Interpolation is a process that determines the unknown z-values of pixels in raster (GRID) form, of a system of points, or in vector (TIN) form of a limited number of points with known data about them. It is used to determine the unknown values at any geographical point of: altitude, precipitation, chemical concentrations, noise level, etc.*

KEY WORDS: *Nonlinear interpolation, Half-Differences, Raster point, Spherical pattern, Exponential model.*

1. Introduction

In linear interpolation, the change of the surface occurs in a linear way. However, sometimes series of values on surface values do not confirm such a linear relationship. In some cases this relationship is more logarithmic, in other cases it is predictable only for small parts of the surface (DeMers, 1989). Under these circumstances, the results of the linear interpolation method will not accurately depict the surface. For this reason, nonlinear interpolation methods have been designed to eliminate the linearity assumption of linear methods. There are several ways to model raster surfaces that use nonlinear interpolation methods. Four of the most commonly used methods are [1,2,3,4]: Inverse Distance Weighted (IDW), Natural Neighbor, Kriging, and Spline. In this article, it is the Kriging method that will be discussed.

The term Kriging comes from Daniel Krige, who developed the method for application in geology. Kriging is one of the most complex and powerful interpolation methods that takes into account the unique characteristics of the data. In this case, concepts and methods of geostatistics are used. This interpolation method assumes that the distances and directions between points reflect a spatial relationship that can be used to explain the diversity of shapes and complexity of the surface. Like the IDW (Inverse Distance Weighted) method, Kriging interpolation is a technique of average weights, with the difference that this method uses more complex mathematics. In the Kriging method, the distances between all possible pairs of points are measured, and this information is used to model the spatial autocorrelation for each individual surface that is interpolated. In other words, Kriging adjusts the calculations to the data we have by analyzing all the points. In this case, Kriging will correspond to a mathematical function for a certain number of points or to all points included for a certain radius. This feature is used to determine the unknown values for each location. It is for the model that best matches the data that the unknown values are predicted. In order to understand how spatial autocorrelation is used in Kriging interpolation, it is necessary to clarify concepts such as half-difference and half-difference diagram beforehand.

The half-difference is determined by the formula:

$$\gamma_h = \frac{\sum_{i=1}^{n-h}(Z_i - Z_{i+h})^2}{2(n - h)} \tag{1}$$

where Zi are the measured values at the points of the surface;

h – the possible distances between the points;

n – number of measured points.

The half-difference calculations in this method require two clarifications. First, since the measured points are usually unevenly distributed over the surface, an assumption is made about the distance and direction between the points. The second assumption to calculate the half-differences is related to the variety of directions – east – west, north – south, northwest – southeast and northeast – southwest.

Let's take a point x as a reference point against which to compare the values of a field in the other locations $z(x_i)$ by increasing the distances from the reference point. We form the differences $z(x) - z(x_i)$. If the field is smooth, the values of adjacent points will not be different, i.e. $z(x)$ will not be very different from $z(x_i)$. To compare the values, we take the difference squared or $\left(z(x) - z(x_i)\right)^2$, the sign is not important. We can do this with any pair of points in the territory.

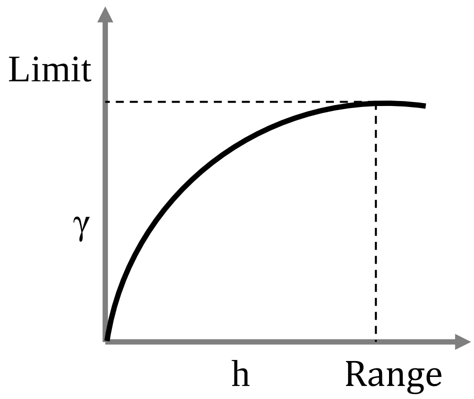

Fig. 1. Half-Difference Chart

The half-difference diagram graphically represents how half-differences change with increasing the value of the distances h. The type of chart with a richer dataset is characterized by the idealized curve shown in Fig.1. Therefore, here we will use it to characterize the elements of the curve and the associated half-differences. Here it is necessary to deliberately use the names of these concepts, as they are found in Western literature, due to their use in software products.

In the figure, the vertical axis represents one half of the square of the differences $z(x) - z(x_i)$, and the graph is known as the half-difference diagram. The distance h is represented along the horizontal axis and represents the distance between the model points, called lag. As is also clear from the figure with an increase in h, i.e. as the model points move away from each other, the half-distances also increase. It is noticeable that the half-differences increase, but in the end they reach a maximum value. This maximum value of the half-difference is called the $limit$, and the distance h at which it occurs is known as $range$. The third important element in the half-difference diagram is that the corresponding curve does not pass through the coordinate origin. Mathematically and conceptually, if there is no distance between the model points, there will be no variance, since the points will be the same point model. But we have to keep in mind that a curve is an estimate of locations. The difference between the expected values at a distance of 0 between the points (lag) is called $nugget$. Burrough (1986) defines this difference as a combination of the residual differences of measurement errors and spatial differences.

The following figure (Fig.2) shows the selection of parameters defining the half-difference diagram with GIS software.

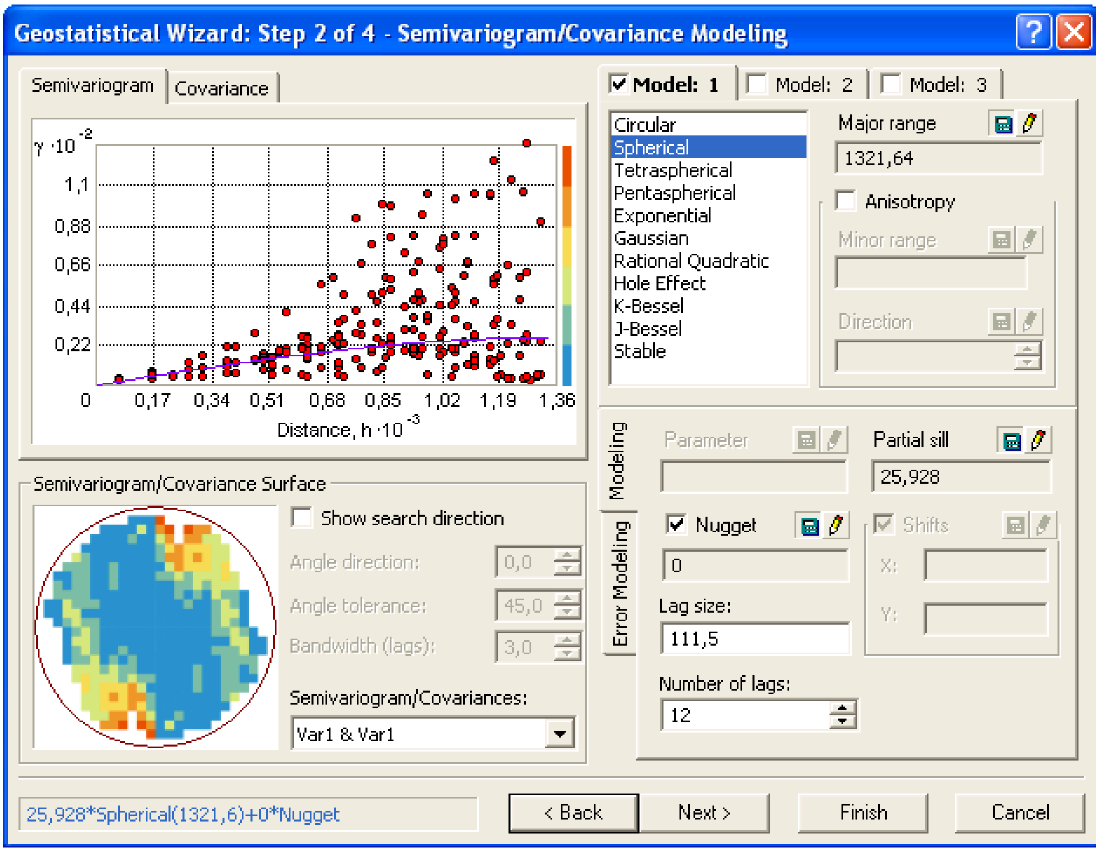

Fig. 2. Kriging Method Half-Difference Diagram

Calculations in Kriging Interpolation

1) ***Calculation of half-differences***: To illustrate the method of interpolation of Fig. 3 depict three measured points and a point x of the presumed raster grid covering a given area. From the coordinates of the points, all possible horizontal distances between the points are calculated, including to the raster point. The data are set out in Tables 1 and 2.

To simplify the calculations, we have entered only three points given with their altitudes to obtain the height of the raster point.

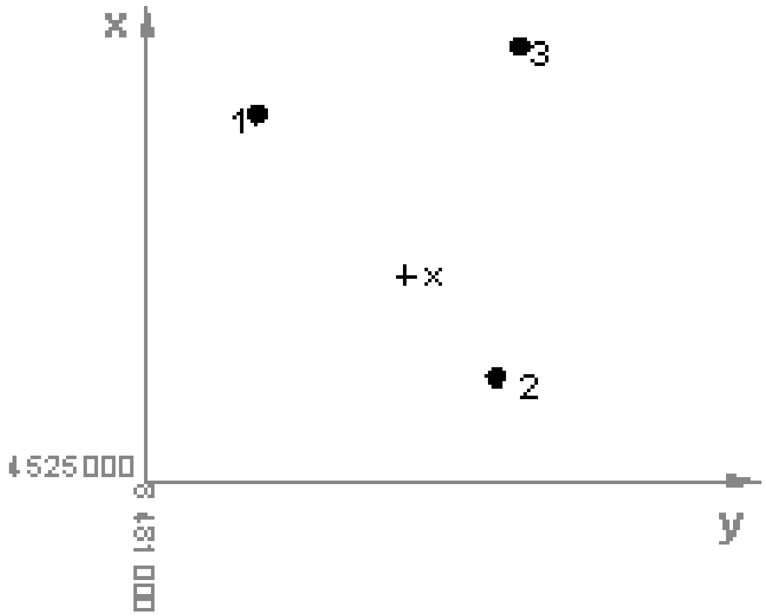

Fig. 3. Graph of model points

Table 1 Coordinate register

№ of point x_i	X	*Y*	*H*
1	4525527.3300	8482751.9300	479.4600
2	4526667.6600	8481494.2900	492.5300
3	4526784.9600	8483077.7400	571.1000
X (raster point)	4526237.5900	8482015.5900	**481.4800**

Table 2 Distances between each pair of points

№ of point x_i	1	2	3	x
1	0			
2	1697.70	0		
3	1302.38	1589.73	0	
X (raster point)	1023.07	675.90	1198.25	0

2) ***Calculating the Raster Point Value***: Kriging interpolation uses average weights to calculate the raster point value. For the three given points, the equation is:

$$\bar{Z} = p_1 Z_1 + p_2 Z_2 + p_3 Z_3, \qquad (2)$$

where $\overline{Z}$ is the value of the raster point;

Z_1, Z_2, Z_3 – the values at the respective measured points;

p_1, p_2, p_3 – the weights associated with each measured point.

The weight p_i is analogous to the value of $1/d$ in the calculations related to weight methods. As is known from regression analysis theory, weights are introduced to minimize the difference between the calculated and actual value of the raster point. Minimization is achieved by applying the Least Squares Method principle or the sum of the squares of the differences $z(x) - z(x_i)$ to be minimal.

In Kriging interpolation, minimization is achieved by determining the weights p_i when solving the following equations:

$$\begin{aligned} p_1\gamma(h_{11}) + p_2\gamma(h_{12}) + p_3\gamma(h_{13}) = \gamma(h_{1x}) \\ p_1\gamma(h_{12}) + p_2\gamma(h_{22}) + p_3\gamma(h_{23}) = \gamma(h_{2x}) \\ p_1\gamma(h_{13}) + p_2\gamma(h_{23}) + p_3\gamma(h_{33}) = \gamma(h_{3x}) \end{aligned} \quad (3)$$

where $\gamma(h_{ij})$ is the half-difference calculated by formula (1) between the measured points i and j, and $\gamma(h_{ix})$ is the half-difference between each $i-th$ measured point and the raster point x.

To illustrate the half-difference calculations, we will elaborate on formula (1) for $\gamma(h_{12})$ and $\gamma(h_{13})$ by entering the calculation data in Table 3.

The solution of equations (3) is usually based on the least squares method. It should be noted that the equations consider not only the distances between known points and the raster point, but also the distances between them. This is the essential difference between the Kriging and IDW (Inverse Distance Weighted) method, which only considers the distances between known points and the raster point.

Table 3 Calculation of half-differences

	h	
	1	2
$(Z_1 - Z_1 + h)^2$	170.82	8397.89
$(Z_2 - Z_2 + h)^2$	6173.24	
$\sum_{i=1}^{n-h}(Z_i - Z_{i+h})^2$	6344.06	8397.89
$2(n-h)$	4	2
γh	1586.01	4198.94

The essential thing here is that an appropriate model is also adopted for the semi-chart associated with a larger data set. GIS provides an opportunity to choose the model of the empirical half-difference diagram: circular, spherical, exponential, Gaussian and linear.

The chosen model affects the determination of unknown values. They are partially dependent on it, especially when the shape of the curve near the coordinate origin differs significantly. A steeper curve near the coordinate origin suggests that closer neighboring points will have a greater impact on the calculation of unknown values. As a result, the resulting surface will not be smooth. Each model is designed to correspond to different types of phenomena with different accuracy. The most commonly used models are linear, spherical and exponential (Fig. 4).

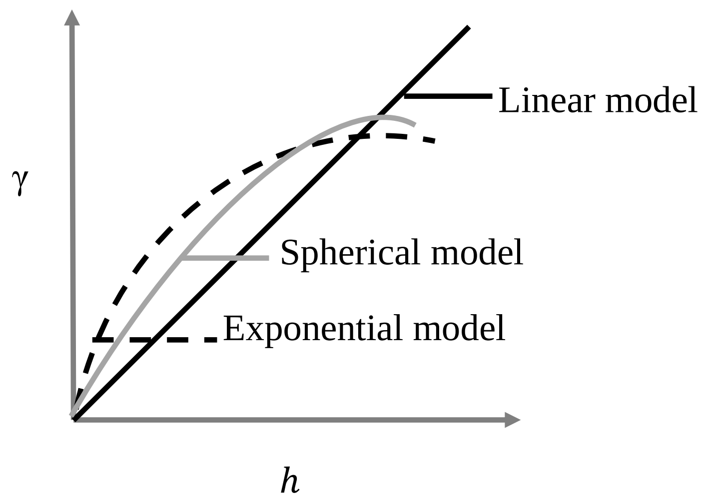

Fig. 4. Different models of the half-difference chart

The spherical model is the most used pattern. This model shows a progressive decrease in spatial autocorrelation. There is an equal increase in half-differences to some distance beyond which the autocorrelation is zero.

The exponential model is applied when spatial autocorrelation decreases exponentially with increasing distance. Let, in our case, assume the linear model as appropriate for the semi-diagram associated with the dataset, from which we have selected three points to determine the unknown raster point. If the model is $\gamma = 10h$, it shows that the half-life is 10 times greater than the distance. In this easy way, the half-differences for each pair of points shown in a table can be calculated, which are entered into the equations (3) to calculate the weights.

They are then entered together with the values of the measured points in the basic formula (2) to calculate the unknown value of the raster point.

2. Conclusion

Although the Kriging method is a more complex interpolation method, it is one of the preferred methods as it gives a relatively accurate measurement of heights for missing values and thus produces more accurate maps. It is only necessary to clearly define the model of the half-difference diagram. Another advantage is that an accuracy assessment is provided for the location of raster points, assuming a normal distribution of errors.

Finally, in Fig. 5 presents the differences in an interpolated surface obtained from a common network of point data in which measurements and different interpolation methods have been made.

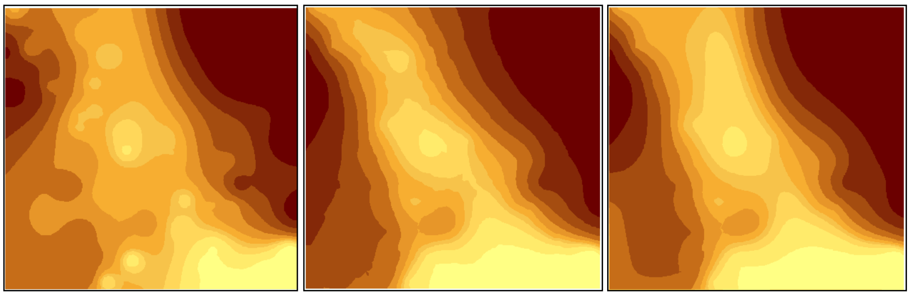

Fig. 5. Results of various interpolation methods:

a) IDW; b) Ordinary Kriging (spherical model); c) Spline

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References:

[1] Kirilova, K., Methodology for establish a local model of geoid (quasigeoid) in mountain and high mountain areas, Annual of Konstantin Preslavsky University of Shumen, Shumen, Konstantin Preslavsky University Press, ISSN 1311-834X, Vol. IX E, 2019, pp. 86-91.

[2] Zhekov, Zh., Antonov, A., Boyanov, P., Chervenkov, D., Trifonov, T., Method for identification of signals, a refereed Journal Scientific and Applied Research (Licensed in EBSCO, USA), Konstantin Preslavsky University Press, ISSN 1314-6289 (Print), ISSN 2815-4622 (Online), vol. 5, 2014, pp. 108-112, DOI: https://doi.org/10.46687/jsar.v5i1.116.

[3] Stoykov, E., Methodology for surveying and tracing of objects by using GNSS“, Annual of Konstantin Preslavsky University of Shumen, Shumen, Konstantin Preslavsky University Press, ISSN 1311-834X, Vol. IX E, 2019, pp. 21-24.

[4] Yanchev, K., Methods for presenting, processing and interpreting the results of geodetic observations of geodynamic processes, Annual of Konstantin Preslavsky University of Shumen, Shumen, Konstantin Preslavsky University Press, ISSN 1311-834X, Vol. IX E, 2019, pp. 55-60.