



## DETERMINATION OF THE ANGULAR ACCURACY WHEN SETTING OUT THE CENTERS OF BRIDGE PIERS

Ani A. Stefanova

UNIVERSITY OF ARCHITECTURE, CIVIL ENGINEERING AND GEODESY, 1 HRISTO SMIRNENSKI BLVD., SOFIA, E-MAIL: stefanova\_fgs@uacg.bg

**ABSTRACT:** The article determines the angular accuracy when setting out the centers of bridge piers, based on a designed geodetic network where the location of the points on the shores is established to ensure the required setting out accuracy of all piers. Theoretical conclusions are carried out under the assumption that setting out of the piers is performed directly by linear-angular measurements from 2 reference points. Strict preliminary assessment is conducted on the accuracy of setting out the centers of bridge piers by measurements from 3 reference points.

**KEY WORDS:** Angular accuracy of setting out, Bridge piers, Horizontal network, Reference points, Straight intersection, Linear-angular measurements, Accuracy assessment.

### 1. Introduction

A key aspect for implementing reliable geodetic assurance in bridge construction is designing a network with a sufficient number of points with an appropriate location, allowing the use of a wide range of modern geodetic methods for horizontal setting out.

The author has developed a method for designing the locations of the reference points from geodetic networks [6] which provides a suitable geometric shape allowing direct setting out of centers of bridge piers with the required accuracy using linear-angular measurements. The presented theoretical conclusions and statements are based on the formulas for assessing the accuracy of a straight intersection (1) [1,2,3,4,5][3].

$$(1) \quad M_p = \frac{m_\beta}{\rho} \cdot \frac{\sqrt{S_{AP}^2 + S_{BP}^2}}{\sin \gamma} = \frac{m_\beta}{\rho} \cdot \Phi,$$

where:  $m_\beta$  – mean square error of a measured/set out angle;  $\beta_1$  and  $\beta_2$  are the measured/set angles at the given points;  $\gamma = 200 - (\beta_1 + \beta_2)$  – the angle at the new

point;  $b$  – the distance between the given points;  $S_{AP}$  и  $S_{BP}$  (Fig. 1) – the lengths between the given and the new point;  $\Phi$  – geometric factor.

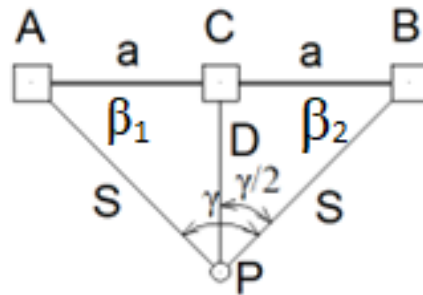


Fig. 1. Part of a scheme of a horizontal network

The reference points of the intersection are located perpendicular to the bridge axis [6].

The developed method is the basis of a methodology for designing geodetic networks [6] (when bridging wide water obstacles) in which the location of points along the shores is established to ensure the required accuracy of setting out all piers, including the most distant one with the possibility to use practically all setting out methods.

## 2. Determination of the angular accuracy of setting out

**2.1. Determination of the angular accuracy  $m_{\beta}^{allow}$  for setting out bridge piers closer to shores ( $P_2$  and  $P_1$ , fig. 2) from the already selected reference points (A, B and D, E) [6].**

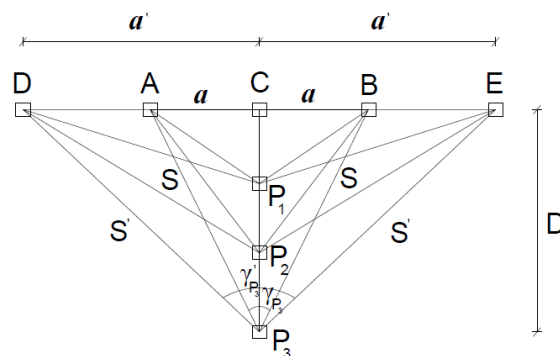


Fig. 2. Part of a horizontal network

Calculations are done in the following way:

- For each bridge pier, the detection angle  $\gamma$  is determined as a function of the distance  $D_i$  from the reference points and the distance „ $a$ “ (Fig. 2) according to formula:

$$(2) \quad \gamma_{P_i} = 2 \arctg \frac{a}{D_i}$$

- Calculation of a parameter „c“ ( $c = \sin \gamma \cos \frac{\gamma}{2}$ ) [6].
- Calculation of  $m_{\beta}^{allow}$  according to formula (3):

$$(3) \quad m_{\beta}^{allow} = \frac{M_P^{allow} \cdot \rho \cdot c}{D_i \cdot \sqrt{2}}$$

where:  $m_{\beta}^{allow}$  - allowable error of a set out angle when measuring/setting out in one set of rounds.  $M_P^{allow}$  – allowable error of setting out;  $D$  - distance from the direction of the reference points to the set out point;  $c$  – coefficient.

These calculations are also done for the other set of reference points D and E. Thus, a double possibility for setting out the bridge piers is provided under the same value of the geometric factor but with a different angular accuracy of setting out  $m_{\beta}^{allow}$ .

Based on the above-described work procedure, a sample diagram of a horizontal network for setting out bridge piers has been developed where the designed location and geometry of “Danube Bridge 2” have been used as output data (Fig. 3).

Applying the above-mentioned work method and observing the sequence of actions described therein, for the most distant pier from the Bulgarian coast which can be set out directly with the required accuracy by linear-angular measurements (straight intersection, linear intersection, bipolar method), pier  $P_4$  has been established. The parameters necessary for determining the locations of the two sets of reference points have been calculated (Fig. 3). Using formula (4), the angular accuracy of setting out  $m_{\beta}^{allow}$  has been calculated for the closer bridge piers (Fig. 3). The values are described in table 1.

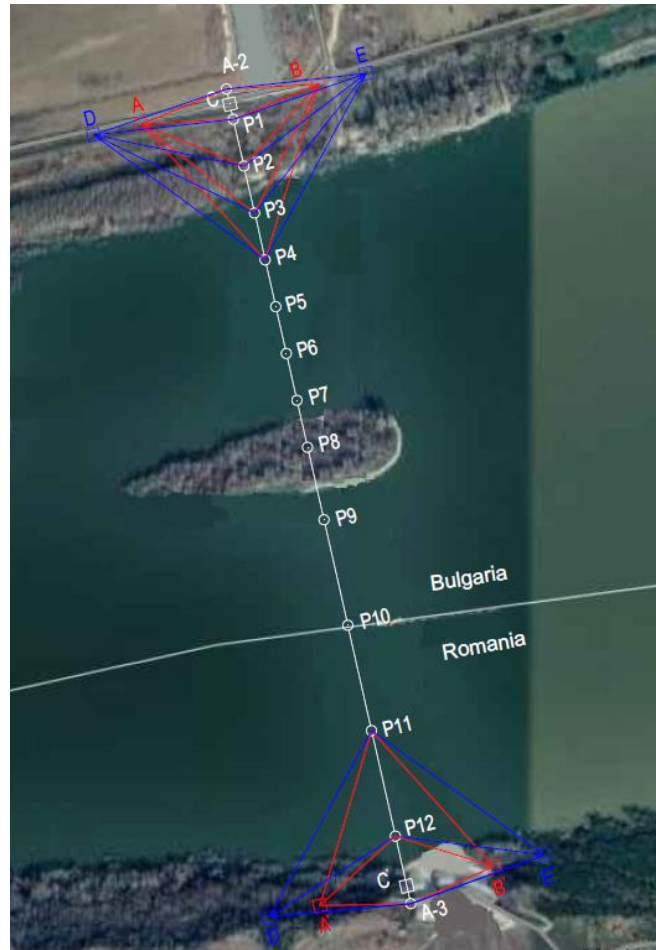


Fig. 3. Sample scheme of “Danube Bridge 2” network

Table 1

Pier №	$D_{CPI}$ [m]	From reference p. A and B				From reference p. D and E			
		$S_{API}=S_{BPI}$ [m]	$\gamma^g$	C	$m_{\beta^{cc}}$	$S_{DPI}=S_{EPI}$ [m]	$\gamma^g$	c	$m_{\beta^{cc}}$
P <sub>4</sub>	266	306	65.6585	0.7464	12	353	91.3805	0.7464	12
P <sub>3</sub>	186	239	86.7327	0.7600	18	298	114.0143	0.6101	14
P <sub>2</sub>	106	184	121.9837	0.5412	22	255	145.4784	0.3137	13
P <sub>1</sub>	26	153	178.2596	0.0569	9	234	185.8043	0.0246	4
A-2	26	153	178.2596	0.0569	9	234	185.8043	0.0246	4

Similar actions were carried out in the design of the part of the sample network on the Romanian coast (Fig. 3) as the most distant support that can be set out directly with the necessary accuracy via linear-angular measurements, is pier P<sub>11</sub>. The locations are determined for the reference points and the angular accuracy of the setting out for the closer piers P<sub>12</sub> and A-3 (Table 2).

Table 2

Pier №	D <sub>CPI</sub> [m]	From reference p. A` and B`				From reference p. D` and E`			
		S <sub>A`Pi</sub> =S <sub>B`Pi</sub> [m]	$\gamma^g$	c	m <sub><math>\beta</math>cc</sub>	S <sub>D`Pi</sub> =S <sub>E`Pi</sub> [m]	$\gamma^g$	c	m <sub><math>\beta</math>cc</sub>
P <sub>11</sub>	265	303	64.9232	0.7436	12	354	92.1587	0.7436	12
P <sub>12</sub>	85	171	133.6966	0.4294	21	250	155.6568	0.2186	11
A-3	30	152	174.3410	0.0785	11	236	183.6247	0.0326	5

The analysis of tables 1 and 2 shows that concerning the closest to the reference points – pier P<sub>1</sub> and abutments A-2 and A-3, are obtained very large detection angles and high requirements for accuracy  $m_{\beta}^{allow} = 4 - 5^{\circ}$  which cannot be achieved in practice by measurements in one set of rounds. In this case, it is recommended that the centers of the mentioned piers to be set out using the polar method.

All theoretical conclusions and research so far have been carried out under the assumption that the setting out of the bridge piers is performed directly via linear-angular measurements from 2 reference points. In order to implement control and possibly to increase the accuracy of setting out, the literature [5] recommends that the setting out is done from three adjacent reference points.

The question here can be stated as: to what extent would the required accuracy of the setting out can be reduced while maintaining the required setting out accuracy  $M_p^{allow}$  using the same end reference points, if one additional midpoint located between them is also used.

For this purpose, a strict preliminary assessment of the accuracy of setting out the centers of the piers in fig. 3 was carried out through linear-angular measurements from 3 reference points using a software product for preliminary assessment of linear-angular networks POCLAM<sup>1</sup>. The assessment was performed iteratively by setting appropriate values of the setting out accuracy  $m_{\beta}$  to ensure a setting out accuracy of the centers of piers close to the required  $M_p^{allow} = 9.5\text{mm}$  [7]. The obtained results are presented in tables 3 and 4. Table 3 presents the piers that will be set out from the reference points A, B, C (D, C, E) on the Bulgarian coast and table 4 – those that will be set out from the reference points located on the Romanian coast.

Table 3

Pier №	From reference p. A, B and C			From reference p. D, C and E		
	M <sub>P</sub> [mm]	m <sub>R</sub> [cc]	m <sub><math>\beta</math></sub> [cc]	M <sub>P</sub> [mm]	m <sub>R</sub> [cc]	m <sub><math>\beta</math></sub> [cc]
P <sub>4</sub>	9.5	9	13	9.4	10	14
P <sub>3</sub>	9.3	14	20	9.5	14	20
P <sub>2</sub>	9.1	23	33	9.2	19	27
P <sub>1</sub>	8.9	37	52	9.3	25	35
A-2	9.0	37	52	9.3	25	35

<sup>1</sup> POCLAM – author: Prof. P. Penev, UACEG

Table 4

Pier №	From reference p. A`, B` and C`			From reference p. D`, C` and E`		
	M <sub>P</sub> [mm]	m <sub>R</sub> [°]	m <sub>β</sub> [°]	M <sub>P</sub> [mm]	m <sub>R</sub> [°]	m <sub>β</sub> [°]
P <sub>11</sub>	9.4	9	13	9.4	10	14
P <sub>12</sub>	9.4	27	38	9.4	21	30
A-3	9.5	38	54	9.5	25	35

Tables 5 and 6 compare the obtained values of the angular accuracy of setting out  $m_\beta$  when setting out the centers of bridge piers from 2 to 3 reference points.

Table 5

Pier №	From reference p. A and B	From reference p. A, B and C	From reference p. D and E	From reference p. D, C and E
	m <sub>β</sub> <sup>cc</sup>	m <sub>β</sub> <sup>cc</sup>	m <sub>β</sub> <sup>cc</sup>	m <sub>β</sub> <sup>cc</sup>
P <sub>4</sub>	12	13	12	14
P <sub>3</sub>	18	20	14	20
P <sub>2</sub>	22	33	13	27
P <sub>1</sub>	9	52	4	35
A-2	9	52	4	35

Table 6

Pier №	From reference p. A` and B`	From reference p. A`, B` and C`	From reference p. D` and E`	From reference p. D`, C` and E`
	m <sub>β</sub> <sup>cc</sup>	m <sub>β</sub> <sup>cc</sup>	m <sub>β</sub> <sup>cc</sup>	m <sub>β</sub> <sup>cc</sup>
P <sub>11</sub>	12	13	12	14
P <sub>12</sub>	21	38	11	30
A-3	11	54	5	35

From the above tables, the following conclusions can be drawn regarding angular accuracy  $m_\beta$  when setting out the bridge piers from 2 and 3 reference points:

- When setting out the more distant piers P<sub>3</sub>, P<sub>4</sub> and P<sub>11</sub> (sight length about 300 m and a distance of the reference points – about 260 m, Fig. 1) from 3 reference points, a decrease in angular accuracy  $m_\beta$  is observed from 6 to 41%.
- When setting out the closer piers – P<sub>2</sub> and P<sub>12</sub> (sight length - 180-250 m and distance of the reference points about 106 m) the angular setting out accuracy decreases significantly, 48 and 82%,

respectively, when setting out from the internal reference points A, B and C (A', B' and C') and by 107 and 170% from the external reference points D, C and E (D', C' and E').

- For the nearest pier  $P_1$  and the abutments A-2 and A-3 (sight length -150-240 m and distance of the reference points about 30m) due to the poor configuration, the required angular setting out accuracy from 2 reference points is very high ( $4''$ ,  $4''$ ,  $5''$ ) and practically cannot be performed with measurements in one set of rounds. However, the situation is different when setting out from 3 reference points – the required setting out angular accuracy significantly decreases (4 to 8 times).

For the remaining bridge piers (Fig. 4) for which the required accuracy cannot be achieved with direct setting out methods performed by linear-angular measurements (straight intersection, linear intersection, bipolar intersection), the use of reduction setting out methods is recommended. For this purpose, it is proposed [6] to design a set of additional reference points on each shore (M, N, M' and N' - Fig. 4) located symmetrically at the midpoint (according to the algorithm in [6]) using a detection angle established for the middle pier of the bridge.

For setting out the centers of the piers, auxiliary points  $P_i^0$  are used located in proximity (5-10 m) to the assumed location of the piers. The coordinates of the auxiliary points are determined with an accuracy higher than the required for setting out by about 10-15%. One-sided or two-sided linear-angular measurements are used towards points of the bridge network with a total station using the “free station” option or GNSS [6]. The setting out of the centers of the bridge piers is done from the auxiliary points via polar method.

In the following calculations, the value used for the required accuracy in the position of the auxiliary point is  $M_{P_0}^{allow} = 8.9mm$  [6].

A strict preliminary accuracy assessment using the sample horizontal network of “Danube bridge 2” (Fig. 4) was performed to derive the required accuracy of angular and linear measurements to determine the position of auxiliary points  $P_i^0$ .

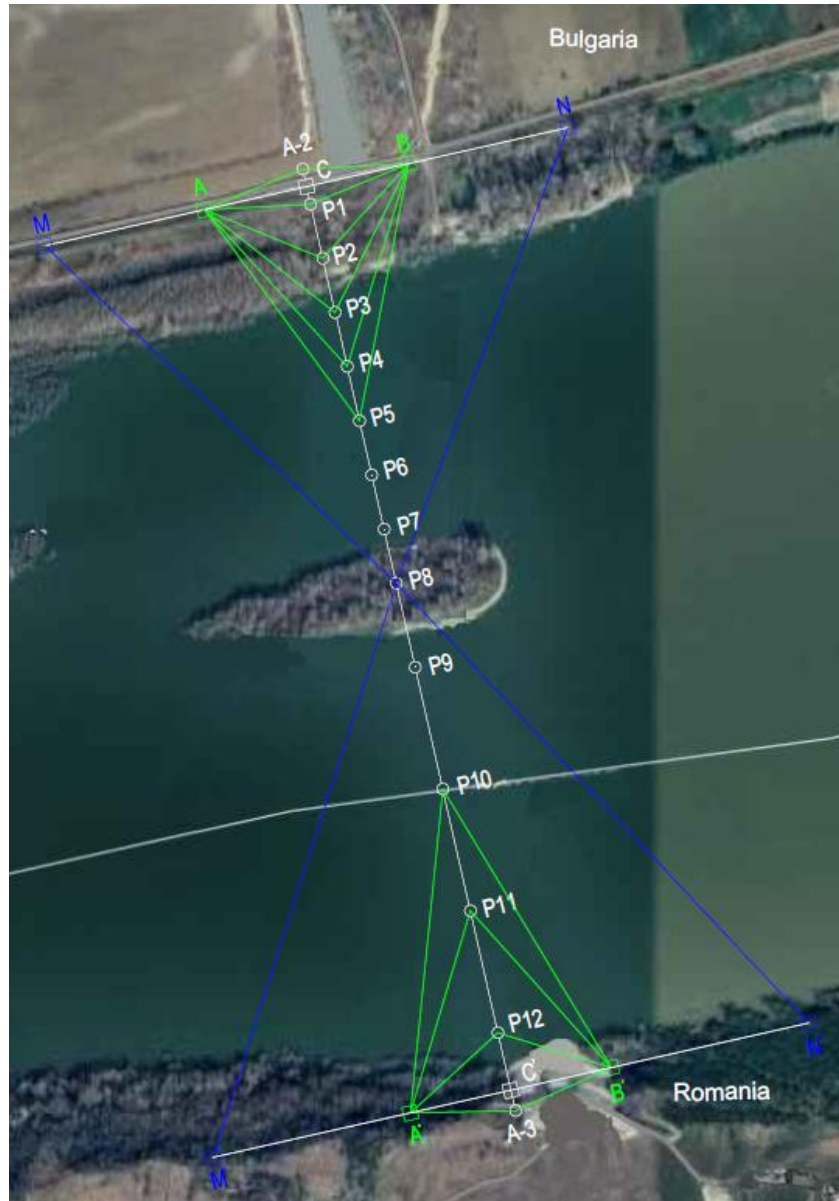


Fig. 4. Sample horizontal network

*Note: The network in Fig. 4 does not show all connections to avoid drawing clutter.*

The assessment was performed iteratively by setting appropriate values for  $m_\beta$  and  $m_s$  to ensure accuracy in the location of the auxiliary points close to the necessary  $M_{P_i^0}^{allow} = 8.9mm$ . The given accuracy of length measurements is  $m_s^{allow} = 8mm/km$ , ( $a=5\text{ mm}$ ,  $b=3\text{ ppm}$ ) since it is assumed that the sights go over a mixed bedding surface (land and water) and the expected temperature variations along the route are more significant, hence their impact on measurement accuracy.



The obtained results are presented in tables 7, 8 and 9.  $S_{aver}$  denotes the average sight length from the reference points to the auxiliary points and  $D_{CP0i}$  – the distance of the auxiliary points from the reference points. Table 7 presents the auxiliary points whose position will be determined one-sided, only from the reference points on the Bulgarian coast and table 8 – those whose position will be determined only from the reference points located on the Romanian coast. Table 9 presents the results for the accuracy in the position of auxiliary points  $P^0_7$ ,  $P^0_8$  и  $P^0_9$  for which it is proposed to be determined from reference points on both shores since these piers are located in the middle of the bridge.

Table 7 One-sided determination from reference points on the Bulgarian coast

Pier №	$D_{CP0i}$ [m]	Average sight length - $S_{aver}$ [m]	From reference p. A, B, C, D, E (S, R)		
			mP [mm]	mR [°]	m $\beta$ [°]
$P^0_5$	346	387	7.1	40	57
$P^0_6$	426	460	8.1	8	11
$P^0_7$	506	535	8.7	3.5	5
$P^0_8$	586	690	8.3	1.5	2.1

Table 8 One-sided determination from reference points on the Romanian coast

Pier №	$D_{CP0i}$ [m]	Average sight length - $S_{aver}$ [m]	From reference p. A', B', C', D', E' (S, R)		
			mP [mm]	mR [°]	m $\beta$ [°]
$P^0_8$	748	769	8.8	0.7	1.0
$P^0_9$	625	649	8.9	1.4	2.0
$P^0_{10}$	445	478	8.1	6	7

Table 9 Two-sided determination from reference points on both shores

Pier №	Average sight length - $S_{aver}$ [m]	From reference p. A, B, C, A', B', C' (S, R)			From reference p. D, C, E, D', C', E' (S, R)		
		mP [mm]	mR [°]	m $\beta$ [°]	mP [mm]	mR [°]	m $\beta$ [°]
$P^0_7$	687	8.8	24	34	8	26	37
$P^0_8$	690	8.7	22	31	8.2	24	34
$P^0_9$	689	8.8	22	31	8.1	23	33

Tables 7 and 8 show that to achieve the required accuracy in the position of auxiliary points  $P^0_7$ ,  $P^0_8$  and  $P^0_9$  ( $M_{P^0}^{allow} = 8.9mm$ ) in one-sided determinations (using reference points on only one or the other shore), the accuracy of angular measurements  $m_{\beta}^{allow}$  is very high (5°, 2.1°/1.0° and 2.0°) and therefore very difficult to reach. This can be explained by the worse geometry due to the one-sided determinations and the bigger average sight length (from 535 to 769 m). This conclusion is confirmed by table 9 which

presents the results of the preliminary assessment for the same points already determined two-sided – from the reference points on both shores. It can be seen that the required angular accuracy (with the above-given longitudinal accuracy – 8 mm/km) significantly decreases to 31-37<sup>cc</sup> and can be easily achieved.

### **Conclusion**

Based on the calculations above, conclusions and recommendations have been drawn concerning the angular accuracy  $m_\beta$  when setting out the centers of bridge piers from 2 and 3 reference points. The conclusions should not be considered as an absolute since the derived accuracies of linear and angular measurements refer to a specific case.

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