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Original Contribution

SURFACE MODELING

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Abstract: *The article introduces us to the types of surfaces, the types of data needed to model these surfaces. The methods for interpolation and modeling of different types of surfaces are also considered.*

Key words: *Statistical surfaces, Continuous surfaces, Interpolation, Extrapolation, Linear Interpolation, Triangulation, TIN Interpolation, Nonlinear Interpolation, Weighted methods.*

1. Introduction

The way we choose to visualize and analyze information depends on how the geographical reality is modeled. Modeling in its essence is the replacement of a real object with its likeness (model) and the study of the model in order to obtain new knowledge about the properties of the object [1]. The geographic database contains four types of data: vector data for the representation of objects; raster data for image representation, thematic data and surfaces; an irregular network of triangles to represent surfaces and addresses to locate locations. With the help of this data, various geographical objects of the surrounding reality can be depicted. With additional operations, they can be edited or new objects can be obtained from them.

2. Exposition

2.1. Statistical surfaces

When quantitative data over a given territory are presented and are mathematically continuously distributed, we say that there is a statistical surface. Two types of data are used to model statistical surfaces. Those that can be obtained by measurement in any possible geographical location are called continuous data. For example, it is known that temperature can be measured at any point on the earth's surface.

Statistical surfaces that use continuous data are continuous [8]. Such a surface is the topographic one, modeled from points with measured heights. With small changes in the statistical information per unit distance, the continuous data present a smooth surface (Fig. 1 (1)). A surface obtained at sharply changing values with a small change in distance is called stepped.

The second type of surface modeling data is the discontinuous ones. Data that occur only in a certain territory, and the values for each unit of territory differ, form discontinuous surfaces (Fig. 1 (2)). The construction of three-dimensional models from the attribute data is based on certain cartographic methods [4]. Statistical surfaces are usually depicted by means of point, line, or area images.



1) Relief 2) Population density

Fig. 1. Continuous smooth and discontinuous stepped surface on the same territory

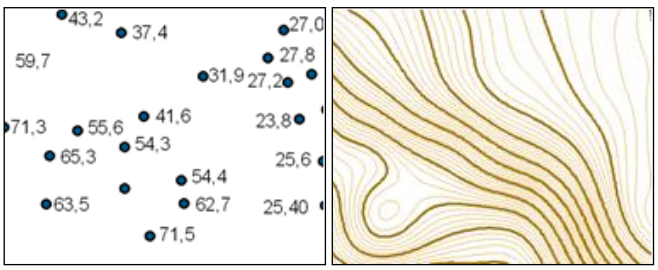
2.2. Analytical methods for modeling continuous surfaces

Typically, a surface contains an infinite number of points. Surfaces can be 2.5D and 3D structured. True 3D models contain multiple z values with the same (x, y) location and thus calculations can be performed in three-dimensional space. When modeling a surface, it is not possible to make measurements at every point of the surface [6]. This applies both to the most common case of modeling a topographic surface, as well as to the modeling of other statistical surfaces from measurements of precipitation, temperature, concentration of pollutants, etc. However, we can obtain individual measurements at each point of the studied territory through an interpolation process. With the appropriate choice of interpolation method, the best approximation to the actual surface can be made.

There are several ways to model three-dimensional data, such as the terrain, geological strata or other surface. Through appropriate algorithms and programs, a surface can be modeled that is defined by a set of points with known coordinates and known connections between the points.

Different ways of vector representation of the surface model are possible (Fig. 2):

- through a system of measured points that can be evenly or unevenly distributed over the surface;
- by contours connecting points with the same quantitative values;
- through a regular (regular) or irregular network of triangles. The TIN structure, as the name suggests, represents the surface as a set of non-overlapping triangles. It is created by a network of points defined by their x, y and z coordinates.



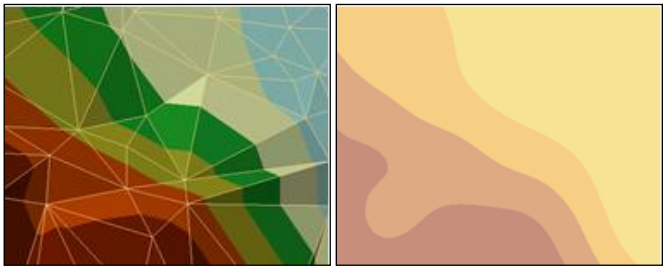


Fig. 2. Imaging a surface via: points with heights, contours, TIN and GRID model

The raster image of the surface in digital form is represented with:

- regular/irregular mosaics of elements (pixels). Most often, pixels are shaped like squares that form a GRID pattern. A grid cell (pixel) stores the z value of the surface at a given location x,y*;*
- TGRID – a model combining TIN and GRID models.

Each of the above approaches to surface modeling has its advantages [5]. TIN networks have exceptional analytical capabilities, and interesting analyses can be performed through raster (GRID) surface models. Only quantitative values of the points do not give a good idea of the shapes of the surfaces, but they are usually used as input data for interpolating raster or TIN surfaces. And the contours will remain the most appropriate means of depicting the relief in topographic maps [7].

The accuracy of the surface depends on the initial data - a certain number of points that define it and their correct distribution over the entire mapped territory. For the accuracy of raster surfaces, the choice and size of cells is important.

2.3. Interpolation Methods for Modeling Continuous Surfaces

What is Interpolation and Extrapolation of Numerical Data?

As is known, a surface can be considered as a continuous field of variable values in the three-dimensional space x, y, z. The coordinate z can be represented as a function of x,y:

$$z = f(x, y). \quad (1)$$

Interpolation is a process that determines the unknown z-values of pixels in raster (GRID) form, of a system of points or in vector (TIN) form of a limited number of points with known data about them. It is used to determine the unknown values at any geographical point of: altitude, precipitation, chemical concentrations, noise level, etc. [10].

A typical case for the use of point interpolation is the creation of a numerical model of the relief obtained from a network of points at which measurements have been made. Each point is represented by its location x and y, and the third coordinate is the obtained altitude h. Unknown values between the entered known points are determined by interpolation. As a result, a network of new values of altitudes at any point on the actual surface is obtained [3]. Unknown values in rasters are obtained by mathematical formulas that use the values of adjacent known points (Fig. 3).

The same principle is used for horizontals, but the interpolation is carried out along the line connecting two adjacent points (i.e. the locations of points with altitudes of integers are determined, e.g. 100 m, 150 m, 200 m, etc.).

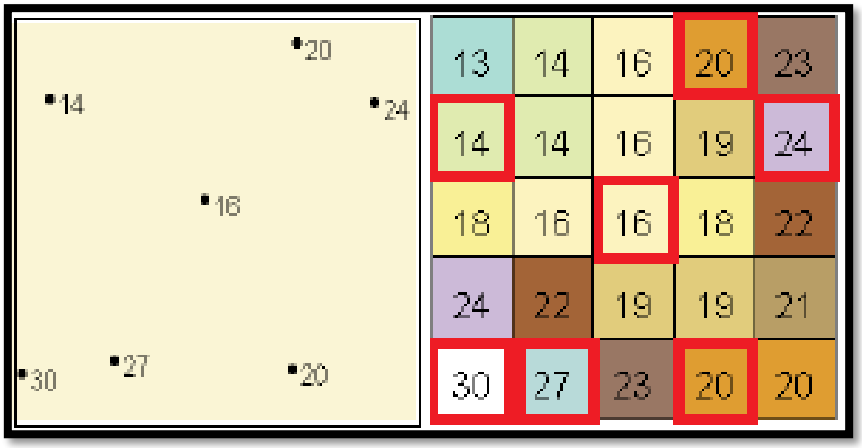


Fig. 3. Interpolation of point data
1). A grid of points with known values; 2). A raster obtained as a result of interpolation from these values.

Interpolation in TIN models consists in determining values corresponding to the geometric centers of the triangles.

Extrapolation is a mathematical method for finding new values of a search function outside its set of known values. As with interpolation, extrapolation replaces the desired function with a similar one, but is usually less accurate, because the values sought are not limited by the known ones. Extrapolation is the only possible method for predicting the values of a time function for future moments in time, due to the inability to find values that cover the searched one.

There are various interpolation methods, such as Inverse Distance Weighted (IDW), Kriging, and Spline, which solve a problem differently, but they all work on the same principle called spatial autocorrelation. In spatial autocorrelation, it is assumed that points that are close to each other are more likely to have close values than those that are distant. Therefore, the Z-value of each individual location must be calculated based on values that are measured at close distances (Fig. 4).

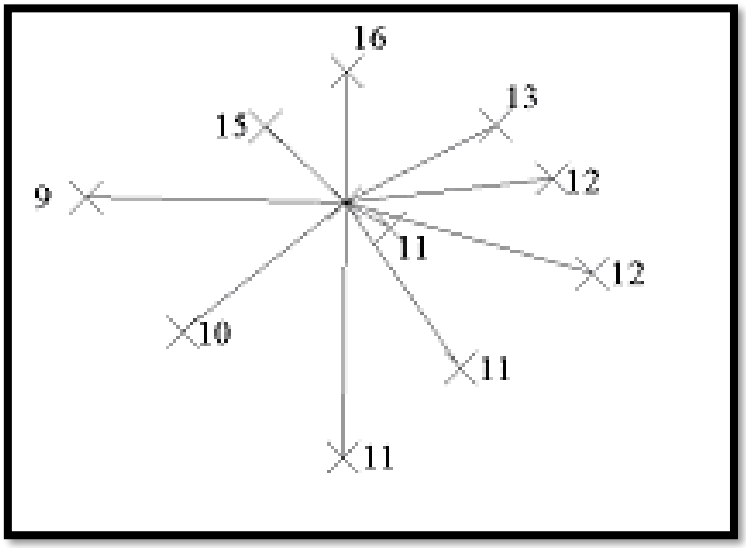


Fig. 4. The unknown value in the center is calculated using the known values of its surrounding points

Linear Interpolation

The interpolation process can be very simple, but it requires a preliminary assumption based on the idea of mathematical progression. Linear interpolation is the process of calculating unknown values that fall between points with known heights spread throughout the territory. The idea here is that the unknown point is proportionally located between the measured points. In the example of Fig. 5, a straight line passes through two points A and B with known values 9 and 10. Between the two measured points is the unknown point M.

$$H_M = H_A + \frac{H_B - H_A}{S_{AB}}.S_{AM} = H_B + \frac{H_A - H_B}{S_{AB}}.S_{BM} \quad (2)$$

If the unknown point is midway between the two points, then the interpolated value is 9.5. If we do this for the entire territory instead of a single line, we can get the integer values for all the contour locations that correspond to the selected intervals in which the surface changes. A characteristic feature of linear interpolation procedures is the production of broken lines that need smoothing. This, of course, involves the use of a second contour smoothing function that the GIS software should have.

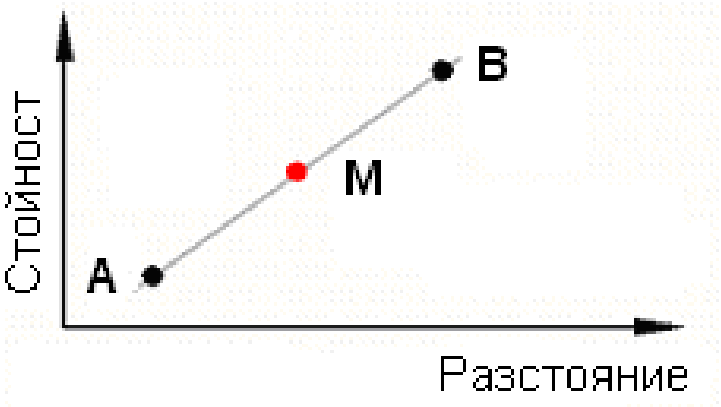


Fig. 5. Linear interpolation between two points

Triangulation. TIN Interpolation

A key point in statistical surface modeling is that the available measured points are incorrectly distributed over the territory. Connected adjacent points form a network of irregular triangles, which are analogous to those of manual interpolation of contours. For this reason, it is logical to consider the triangulation approach as the first interpolation method, as it suggests how the contours were made manually. Although manual interpolation is not practiced today, it is still important and is often used as a criterion against which to compare automated methods.

If we have three points with coordinates Xi,Yi,Hi (I = 1,2,3) and the coordinates of an inner point M, we find the height of HM from the equation of the plane passing through the points:

$$H_M = A.X_M + B.Y_M + C, \quad (3)$$

Where the coefficients A, B, C are calculated from the following three equations:

$$A.X_1 + B.Y_1 + C = H_1$$
$$A.X_2 + B.Y_2 + C = H_2$$
$$A.X_3 + B.Y_3 + C = H_3 \quad (4)$$

John Davis (2002) points out that a problem in triangulation is determining the best grid of triangles. A suitable solution is to find the closest adjacent points, and this is usually the first step in interpolation, in both manual and automated methods. This problem disappears with the construction of Delaunay triangles.

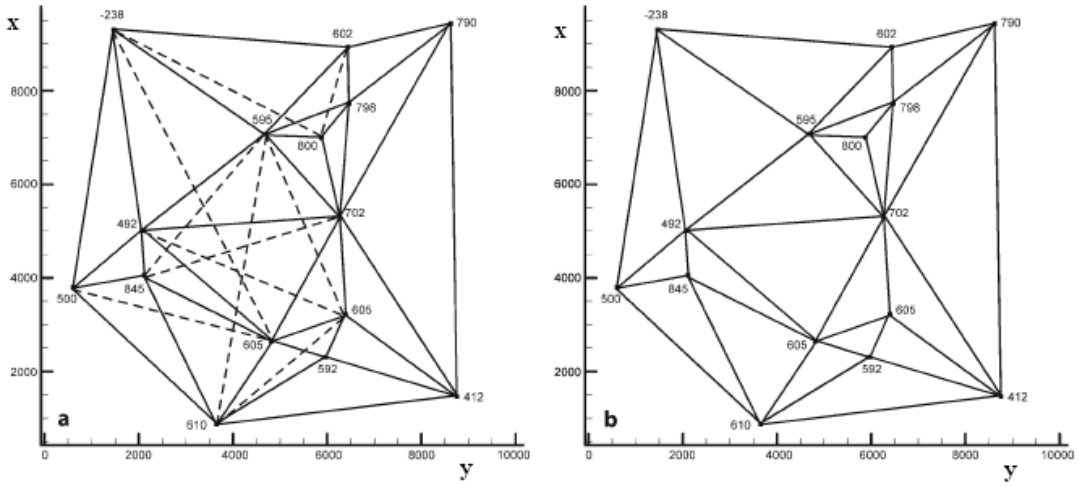


Fig. 6. Nearest Adjacent Points in a TIN Network
A) Alternate close adjacent points; B) Longer (interrupted) sides have been removed to define the nearest adjacent points.

Such a triangle is obtained if no other points fall in the vicinity of the circle passing through the three points that form the triangle. Then the vertices of the triangle are the closest adjacent points. An appropriate method for selecting adjacent points is by finding the shortest distance between the vertices of the formed triangles (Fig. 6). Delaunay triangles are closely related to Thiessen polygons (Fig. 7), which are formed by drawing boundaries between the measured points, so that all hypothetical points in the polygon are closer to the measured point located inside the polygon than any other measured point.

The TIN model is the basic vector form for rendering a statistical surface using a computer, based on the triangular interpolation method. TIN is one of the many methods of storing elevation information to create a numerical elevation model, and is known to be based on a system of points with known elevations. This system of points forms a network of irregular triangles, which subsequently serves as the basis of a regular grid, whose values are derived from the known point data. This process is called TIN interpolation, which is also an analytical method. Subsequently, the TIN model can be converted to a raster surface.

A grid of triangles is used to store data on linear objects that play an important role in surface modeling, such as watershed and spillway lines. Because the points can be arranged unevenly, the TIN model can be of higher resolution, when the surface is more folded, then more points are required to render more details. In the case of a flat territory, a lower-resolution TIN is selected. TIN models are used less compared to raster surface models due to

their complex data structure, but they are still used to model some high-precision engineering applications.

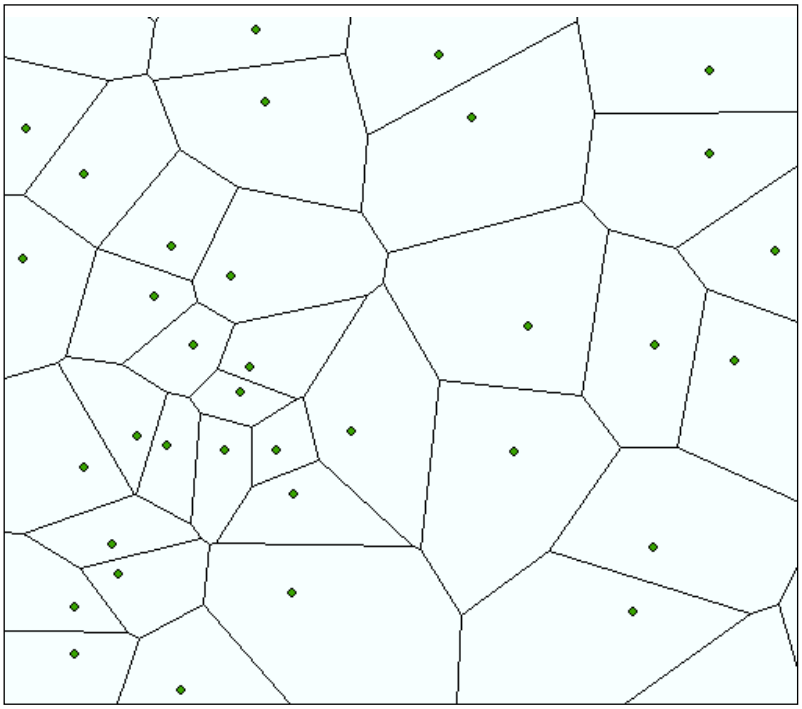


Fig. 7. Thiessen Polygons

Nonlinear Interpolation

In linear interpolation, we assumed that the change of the surface occurs in a linear manner. Sometimes, however, the series of values on the surface does not confirm such a linear relationship. In some cases, this relationship is more logarithmic, in other cases it is predictable only for small parts of the surface [2]. Under these circumstances, the results of the linear interpolation method will not accurately depict the surface. For this reason, nonlinear interpolation methods have been designed to eliminate the assumption of linearity in linear methods. There are several ways to model raster surfaces that use nonlinear interpolation methods. Four of the most commonly used methods are: Inverse Distance Weighted (IDW), Natural Neighbor, Kriging and Spline.

Nonlinear spatial interpolation calculates the unknown values using a set of points with known values that are spread over the entire area of the territory under consideration. The distance from a point with an unknown value to a point with a known value helps to determine the unknown value.

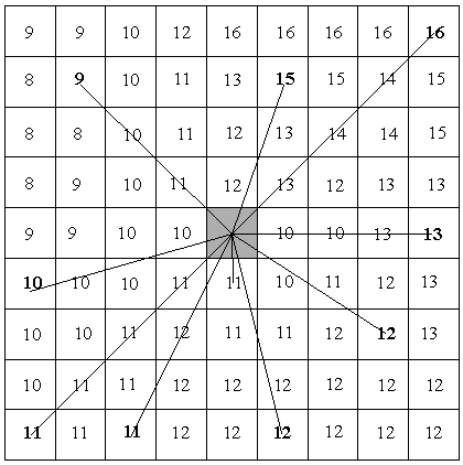
Spatial interpolation is used to create a surface with a certain number of points. With a larger number of points, a more detailed representation of the surface is provided [11]. Points of some value should be evenly distributed throughout the mapped area. However, some locations may require a higher number of points to more accurately represent sudden changes in the character of the surface. For example, determining the size and shape of a hill requires a significant number of points to obtain an accurate model, while in flat areas a smaller number of points is required.

We have mentioned that there is the concept of spatial autocorrelation. The principle on which spatial autocorrelation is based, formulated by Waldo Tobler, states that, each point in space is related to another point, giving more importance to more closely spaced points than to those that are at a greater distance. Most interpolation methods allow control over the

number of points with known values that are used in calculating the value of the unknown raster cell, using the closest ones and ignoring the rest.

Weighted methods

There are several weighted methods, but they all derive from the most popular in Western literature, known as Inverse Distance Weighted (IDW). The term "*weight, inversely proportional to distance*" is used because the measured elevation points have weights that are the inverse function of their distance to points of unknown heights. Since a grid grid is used, this interpolation method is often also found under the name *gridding*. The method has no analogue to manual interpolation methods. This method of interpolation involves three steps: 1) covering the measured elevation points with a grid grid; 2) calculating the values at each raster point of the grid as a function of the distance to the measured points, and 3) interpolating between the points (vertices) of the regular raster grid to create contours.



Each point i has a known height Zi, location (Xi, Yi), weight pi and distance di to raster cell x.

Fig. 8. Calculation of raster cells

The unknown value of the raster cell is determined by points with known values and the relative distance of the cell to these points.

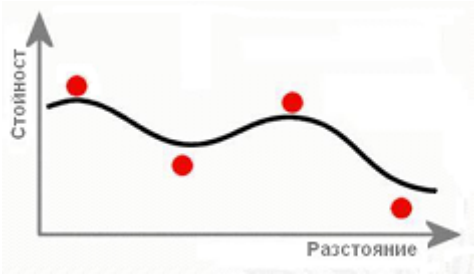


Fig. 9. The surface created by the IDW interpolation method does not pass through the measured points

Weighting methods assume that the points closer in location have closer values and are more dependent on each other. For example, when we go to the top, we notice that the altitudes near us are almost the same as for the distant points. In the same way, when we descend from the top, the change in altitude values for neighboring points will be similar [12].

At the foot of the hill, however, we notice that the altitude values change quite quickly, and in the plane after the base of the hill, uniformity in altitudes suddenly appears again. On the way of this reasoning, the measured points near the raster point have greater weights than the distant ones.

Let's mark the point we are interested in with x and the points where measurements with xi have been made, where i changes from 1 to n if there are n points. We mark the unknown value with Z(x) and the known measurements with Zi. We give each of these points a weight pi, which is calculated on the basis of the distance di from the desired point xi to each point x. Fig. 8 explains this notation.

The basic formula for calculating the raster point value is:

$$Z(x) = \sum_{i=1}^{n} p_i Z_i \Big/ \sum_{i=1}^{n} p_i \quad . \tag{5}$$

In other words, the interpolated value is one average-weight of the observed values with a weight of pi. There are different ways to calculate weights, but the most commonly used is to calculate them as inversely proportional to the square of the distances or

$$p_i = 1 / d_i^2 \quad . \tag{6}$$

Most software products allow the user to use several strategies in determining the number of measured points for calculating the values of raster points (Fig. 10).

Some of these strategies can use all measured points in calculating each raster point, others in ignoring all points that are distant at some particular distance, or in limiting a certain number of nearest points, or in averaging the nearest points in each of the ruling sectors. The effect of these strategies has been discussed and described in detail by Terry Slocum and team [9].

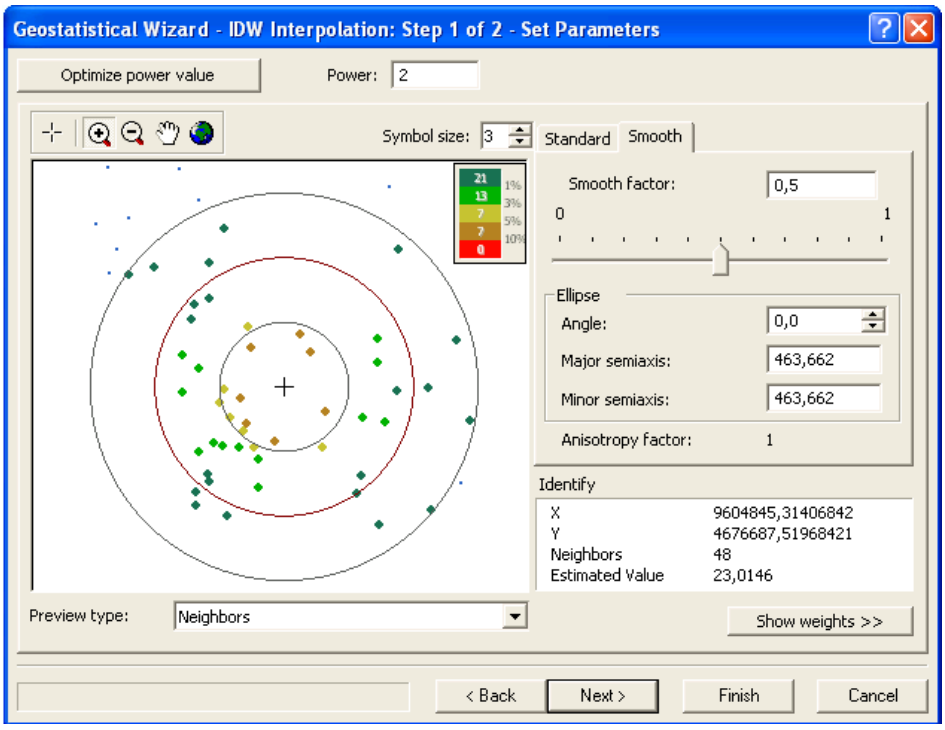


Fig. 10. Selecting measured points participating in IDW interpolation with ArcGIS

Finally, we will point out some advantages and disadvantages of the weighted interpolation method. The method is a simple way to calculate values from a field with locations where measurements are not available. Interpolation achieves the desired goal of creating a smooth surface, whose values at each point are more similar to the values at adjacent points than to the values at distant points.

As with triangles, simple linear interpolation results in broken contour lines. This problem can be eliminated by increasing the number of raster points (this in turn leads to an increase in file size and requires more computer memory) or by smoothing the lines. Smoothing is done through the splining function, which models a relatively smooth surface. Often in software products, this method is also used as a standalone method of interpolation.

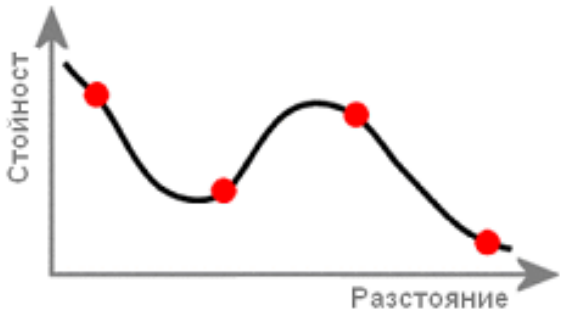


Fig. 11. Surface obtained by the Spline interpolation method

Instead of averaged values, as is the case with IDW, the Spline interpolation method calculates unknown values by bending a single surface that passes through all known values of the points. This concept is illustrated graphically in Fig. 11.

We can imagine that we are stretching the surface that passes through all points with known values. This stretching effect is useful if we want to calculate values that are below or above the maximum values found in the list of data we have. This makes this interpolation method good for calculating maximum and minimum values when they are not included in the data with known values. This method is best suited for smoothly changing surfaces, such as the topographic surface. However, when the points are located close to each other and have significant differences in their values, the Spline interpolation is not very suitable. For example, geographic features such as rocks or fault lines where there is a sudden change in surface values are not very suitable for modeling in this case. Then it is preferable to use IDW interpolation, where the possibilities of modeling "barriers" can be used to overcome sudden changes in values.

There are two types of Spline methods in software products: Regularized and Tension. For the same series of points, the Tension Spline method produces a more bending surface, while the Regularized Spline method creates a more elastic surface.

Another way to improve the results of the weighting method and to form itself as a standalone method is the Natural Neighbor method. In this method, the unknown value for a location is calculated similarly to the IDW method, i.e. as the average weight of the values of neighboring points. This method of interpolation can effectively deal with a large number of points. From all the points entered, a triangulation is created and the nearest vertices are selected, which form convex triangles around the interpolated point [13]. Then the weights are proportional to their areas. This method is most suitable when the point data are distributed with uneven density.

5. Conclusion

There is a problem with weighted interpolation where the direction of the data cannot be determined. For example, if a raster point can be visibly assigned a higher value than the values of the measured points, then the IDW method cannot offer a higher or lower value than any of these points. This problem exists because the formula for the method is the total mean of the z values of the measured points, and the total mean of the z values cannot be greater or less than any value entered. For this reason, IDW interpolation can produce inverse-intuitive results for a territory that contains vertices and depressions.

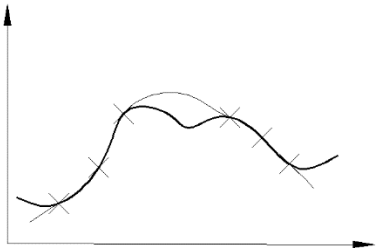


Fig. 12. Possible undesirable outcomes of IDW interpolation for territories where insufficient data are insufficient

Let's imagine an altitude surface with peaks and depressions, and assume that the peaks and indentations are not actually measured. Fig. 12 shows a cross-section of such a surface. Instead of interpolating the peaks and indentations, as one would expect, the IDW gives the result shown in the figure – small indentations where the vertices should be, and vice versa. This behavior is often observed when using the IDW method.

In short, weighted results are not always what we want them to be. There are much better spatial interpolation methods that solve the problems that have just been identified. However, the convenience of using the weighted method (IDW) is its conceptual simplicity, and this makes it among the most popular interpolation methods. Users of GIS software products should just be careful and be sure of the results of the interpolation.

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