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Original Contribution

EVALUATING AN INTERACTIVE LECTURE MODEL FOR TEACHING COMPLEX NUMBERS: A COMPARISON WITH TRADITIONAL INSTRUCTION

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ABSTRACT: *In this paper, an interactive lecture model for teaching complex numbers through coordinated algebraic, geometric, and trigonometric representations is examined. Two existing groups of first-year engineering students received traditional (n = 17) or interactive instruction (n = 24). A common post-test was administered immediately after the 45-minute lecture, followed by a questionnaire in the interactive group.*

A higher overall mean score was obtained by the interactive group (5.92 versus 4.12 out of 10; p = 0.035, Hedges' g = 0.66). For the six conversion and coordination questions, the difference was not statistically significant (3.38 versus 2.47 out of 6; p = 0.117). Higher performance was also observed on within-representation questions; therefore, a representation-specific advantage was not established. Attention and active involvement were generally reported, while understanding of relationships among representations was less consistently endorsed.

Higher immediate achievement was associated with interactive instruction. Causal conclusions and evidence of lasting differences are limited by non-random group assignment and assessment after a single lecture.

Key words: *Complex numbers, Interactive lecture, Multiple representations; Mathematics education, Engineering students.*

1. Introduction

Complex numbers are an essential component of university mathematics and provide mathematical tools for areas such as electrical engineering, signal processing, control systems, and applied physics. Their study introduces students to a number system in which algebraic operations, geometric objects, and trigonometric relations describe the same mathematical entity. For first-year engineering students, learning this topic therefore requires more than applying computational rules. Students must recognize a complex number in different forms, move between these forms, and select a representation appropriate to the problem being solved.

Research indicates that these abilities do not necessarily develop simultaneously. Students may perform algebraic procedures correctly while experiencing difficulty in

interpreting a complex number as a point or vector, determining its modulus and argument, or relating its Cartesian and trigonometric forms [8, 10, 13]. Consequently, the representations may be treated as separate pieces of knowledge rather than as coordinated descriptions of the same mathematical object.

The present study builds on an interactive lecture model for teaching complex numbers through coordinated representations proposed by Todorova-Lazarova [14]. The model is informed by Duval's distinction between treatment within a representation and conversion between representations [2], together with the mental constructions described by APOS theory [1]. Previous studies have compared conventional and GeoGebra-supported instruction in complex numbers among undergraduate engineering students [11, 12]. However, the coordinated use of algebraic, geometric, and trigonometric representations has not clearly been placed at the centre of the assessment in these comparisons. There is also limited evidence concerning students' perceptions of a short interactive lecture structured around these theoretical perspectives.

The aim of the study is to compare the outcomes of a traditional textbook-based lecture and a content-matched interactive lecture on complex numbers. Two existing groups of first-year engineering students completed the same diagnostic prerequisite test immediately before instruction and the same complex-number post-test immediately after instruction. The analysis compared overall post-test achievement and performance on tasks requiring conversion and coordination among representations. Students in the interactive group also completed a questionnaire concerning their experience of the lecture. The study contributes a comparison of immediate overall achievement and a separately reported outcome for questions involving conversion and coordination among representations, together with students' perceptions of the interactive lecture.

2. Related work

Research on the learning of complex numbers has repeatedly identified difficulties associated with their different forms. Panaoura et al. examined geometric and algebraic approaches and showed that success in one form does not necessarily imply flexible movement to another [10]. Karakok, Soto-Johnson, and Dyben similarly found that different forms of complex numbers may be understood unevenly and that transitions between them require more than knowledge of the corresponding formulas [7].

Studies involving university students provide further evidence of this difficulty. Smith, Zwolak, and Manogue analyzed how physics students coordinated forms and representations in complex algebra [13]. Successful students tended to maintain a coherent strategy, whereas unsuccessful solutions often combined incompatible trigonometric approaches. Kin Eng and Fui Fong distinguished between knowing procedures and grasping the mathematical object represented by them [8]. Their analysis showed that procedural knowledge does not by itself ensure a connected understanding of complex numbers. Mutambara and Tsakeni also identified cognitive obstacles among undergraduate physics student-teachers, particularly in relation to division and polar form [9].

Recent work has examined how meaning develops for specific forms of complex numbers. Karagöz Akar, Saraç, and Belin studied prospective teachers' development of the Cartesian form and demonstrated the importance of relating its components to quantities and geometric meaning [6]. Karagöz Akar et al. subsequently investigated teachers' knowledge of different forms through quantitative reasoning [5]. The results indicate that knowledge of individual forms and the ability to coordinate them are distinct aspects of mathematical understanding.

Duval's theory of semiotic representation registers provides a direct explanation of this distinction [2]. A treatment is a transformation carried out within the same register, such as multiplying two complex numbers in algebraic form. A conversion changes the register while preserving the mathematical object, such as transforming an algebraic expression into a point in the complex plane or into trigonometric form. According to this perspective, mathematical understanding requires the coordination of at least two registers. For complex numbers, this means that correct symbolic manipulation alone is insufficient if the student cannot connect it with geometric and trigonometric meaning.

APOS theory provides a complementary account of how mathematical ideas are constructed through actions, processes, objects, and schemas [1]. An action may initially depend on an explicit rule or external prompt. Through reflection and repeated use, it may be interiorized as a process and subsequently understood as an object that can be transformed and related to other objects. In complex-number instruction, this perspective can guide the progression from carrying out isolated procedures to recognizing relationships among algebraic expressions, points or vectors, moduli, arguments, and trigonometric forms. Karakok et al. [7] and Mutambara and Tsakeni [9] demonstrate the relevance of process–object development to the learning of complex numbers. However, APOS has been used more often to interpret understanding than to structure and evaluate a short comparative lecture intervention.

Dynamic mathematical tools can make relations among representations visible, but their educational effect depends on the design of the learning activities. The meta-analysis by Juandi et al. reported generally positive effects of GeoGebra-supported mathematics learning while also identifying variation across settings and instructional conditions [4]. Hillmayr et al. reached a similar conclusion in a broader meta-analysis of digital tools in mathematics and science education: technology is most effective when its use is aligned with suitable pedagogy and subject-specific tasks [3]. These findings do not support treating software use as an instructional method in itself. Instead, dynamic visualisation should serve a defined mathematical and pedagogical purpose.

The studies by Seloane, Ramaila, and Ndlovu are the closest direct comparisons with the present research. Their 2023 study compared GeoGebra-supported and conventional instruction among undergraduate engineering mathematics students and examined students' knowledge of complex numbers [12]. Their 2025 study focused on misconceptions and errors and reported an advantage for GeoGebra-supported instruction in reducing their overall frequency [11]. These studies establish that comparisons between conventional and technology-supported teaching of complex numbers already exist, including research with engineering students. Therefore, the gap cannot be defined as an absence of such comparisons.

The remaining issue concerns the theoretical structure of the intervention and the outcomes selected for assessment. Existing direct comparisons are grounded primarily in other instructional perspectives and focus mainly on overall achievement, procedural and conceptual knowledge, or frequencies of errors and misconceptions [11, 12]. The present study extends these comparisons by separately examining performance on a set of questions requiring conversion or coordination among algebraic, geometric, and trigonometric representations. It also examines students' perceptions of a short interactive lecture informed by Duval's treatment–conversion distinction and APOS theory.

3. Experiment

3.1. Research design

The study employed a quasi-experimental design with two existing groups of first-year engineering students. Group A received a traditional lecture, whereas Group B participated in an interactive lecture. Both groups completed the same diagnostic test immediately before instruction and the same achievement test immediately after instruction.

The instructional conditions covered the same mathematical content, pursued the same learning objectives, and had the same duration. The principal difference concerned the mode of instruction. The study examined whether the interactive approach was associated with higher overall achievement and better performance on tasks requiring conversion and coordination among algebraic, geometric, and trigonometric representations of complex numbers.

The following research questions guided the study:

1. Are the two groups comparable in terms of the prerequisite mathematical knowledge assessed before instruction?
2. Is there a difference between the traditional-lecture group and the interactive-lecture group in their overall performance on the common post-test?
3. Is there a difference between the groups in tasks requiring conversion and coordination among algebraic, geometric, and trigonometric representations?
4. How do students in the interactive-lecture group perceive the support provided by visualisations, their attention and active involvement, and their understanding of the relationships among representations?

3.2. Participants

The participants were 41 first-year students at the Technical University of Gabrovo, Bulgaria. Group A consisted of 17 students, and Group B consisted of 24 students. Each group included students enrolled in different engineering programmes within the Faculty of Electrical Engineering and Electronics and the Faculty of Mechanical and Precision Engineering.

The groups followed the existing organization of the educational process. The students were therefore not randomly assigned to the two instructional conditions. This characteristic determined the quasi-experimental nature of the study.

Only students who completed both tests and participated in the corresponding lecture were included in the comparative analysis.

3.3. Instructional approaches

Both lectures addressed the algebraic form of a complex number, real and imaginary parts, geometric representation in the complex plane, modulus and argument, trigonometric form, operations with complex numbers, De Moivre’s formula, and roots of complex numbers. Each lecture lasted 45 minutes.

The traditional lecture followed the standard textbook-based sequence used in first-year engineering mathematics courses. The definitions and properties were presented by the lecturer and illustrated through worked examples. The students primarily followed the exposition and took notes. No planned interactive activities or dynamic visualisations were included.

The interactive lecture covered the same mathematical content but organised it through guided questions, short student tasks, prediction, discussion, and explicit transitions among representations. Students were encouraged to relate an algebraic expression to its corresponding point or vector in the complex plane and to its trigonometric form. They were

also asked to interpret multiplication, powers, and roots in both symbolic and geometric terms.

The interactive design was informed by Duval's distinction between treatment within a representation register and conversion between registers. APOS theory informed the sequencing of activities from performing mathematical actions to recognising the underlying processes and relationships. GeoGebra was used for dynamic visualisation when it supported these activities, but it was treated as an instructional tool rather than as the defining component of the intervention.

3.4. Research instruments

Three instruments were used: a diagnostic prerequisite test, a complex-number post-test, and a student questionnaire.

The diagnostic test assessed knowledge considered necessary for learning complex numbers, including real numbers, Cartesian coordinates, the Pythagorean theorem, basic trigonometry, and algebraic operations. The maximum score was 20 points. Its purpose was to describe and compare the two groups' performance on the assessed prerequisite knowledge before instruction. It was not designed as a parallel form of the post-test.

The post-test consisted of ten multiple-choice questions and was identical for both groups. Each correct answer was awarded one point, producing a total score between 0 and 10. Three questions assessed operations within the algebraic or trigonometric form, six required conversion or coordination among algebraic, geometric, and trigonometric forms, and one assessed knowledge of the number of complex roots. The six conversion and coordination questions produced an additional subscore ranging from 0 to 6.

The post-test included conversions from algebraic to geometric and trigonometric representations, from trigonometric to algebraic and geometric representations, and from a geometric representation to coordinated algebraic and trigonometric forms. One question required students to connect all three representations of the same complex number.

Following the post-test, students in Group B completed a questionnaire about their experience of the interactive lecture. The four statements asked whether the visualisations supported their understanding, the lecture maintained their attention, they understood the relationships among the different representations, and they felt actively involved.

3.5. Procedure

The topic of complex numbers was the first topic in the relevant course plan. The diagnostic test was administered immediately before the lecture and required approximately 15 minutes. Group A then attended the traditional lecture, while Group B attended the interactive lecture.

Immediately after the lecture, both groups completed the same post-test within approximately 15 minutes. The students in Group B subsequently completed the questionnaire about the interactive lecture.

Thus, the main procedure for both groups followed the same sequence and lasted approximately 75 minutes.

3.6. Data analysis

The diagnostic-test results were analyzed descriptively to compare the two groups' performance on the assessed prerequisite knowledge. Because the diagnostic and post-instruction tests measured different constructs, a learning-gain score obtained by subtracting the diagnostic score from the post-test score was not calculated.

The primary outcome was the total post-test score. The conversion and coordination subscore was treated as a secondary outcome. Descriptive statistics were calculated for both groups. Between-group comparisons were conducted using Welch's independent-samples t-test. Hedges' g was reported as an effect size for the total post-test score.

Responses to the questionnaire were analyzed descriptively and were used to interpret how the students experienced the interactive lecture. Because the questionnaire was administered only to Group B, its results were not used for a direct comparison of student satisfaction between the two instructional conditions.

4. Results

4.1. Prerequisite knowledge

The diagnostic test assessed mathematical knowledge required for learning complex numbers. The traditional-lecture group obtained a mean score of 12.00 out of 20, and the interactive-lecture group obtained 11.75 out of 20. The reported mean diagnostic-test scores were close; this descriptive comparison does not establish equivalence between the groups. The diagnostic and post-instruction tests measured different content; therefore, their scores were analyzed separately, and no learning-gain score was calculated by subtracting one from the other.

4.2. Overall post-test performance

The interactive-lecture group obtained a higher mean score on the common post-test administered immediately after instruction (Table 1). Its mean was 5.92 out of 10, compared with 4.12 for the traditional-lecture group. The difference was 1.80 points. On the six-point scale, calculated as $\frac{Score}{10} \cdot 4 + 2$, the corresponding mean grades were 4.37 and 3.65.

Table 1. Overall post-test performance

Group	n	Mean score / 10	SD	Mean grade
Traditional lecture	17	4.12	2.36	3.69
Interactive lecture	24	5.92	2.83	4.37

Note: SD = standard deviation.

Mean grades were calculated from unrounded mean scores.

A Welch independent-samples test showed a difference in total post-test scores, $t(37.50) = 2.19,\ p = 0.035$. The estimated mean difference was 1.80 points, with a 95% confidence interval from 0.13 to 3.47 points. The effect size was Hedges' $g = 0.66$. These results describe achievement measured immediately after the lectures.

4.3. Conversion and coordination of representations

Six post-test questions required students to convert between or coordinate algebraic, geometric, and trigonometric representations. The interactive-lecture group obtained a mean of 3.38 correct answers out of six, compared with 2.47 for the traditional-lecture group (Table 2).

Table 2. Performance on the six representation questions

Group	n	Mean / 6	SD
Traditional lecture	17	2.47	1.70
Interactive lecture	24	3.38	1.88

The mean difference, calculated from unrounded scores, was 0.90 points in favour of the interactive-lecture group. For this subscore, the Welch test yielded $t(36.65) = 1.60,\ p =$

0.117. Thus, the representation-question results show the same direction as the overall result, but the evidence is insufficient to conclude that this secondary difference is statistically significant at the 0.05 level.

On the three within-representation processing questions, the interactive-lecture group obtained a mean of 2.04 correct answers out of three, compared with 1.35 for the traditional-lecture group, a difference of 0.69 points. The observed advantage was therefore not confined to the conversion and coordination questions. These descriptive results do not establish a stronger effect specifically on conversion and coordination.

4.4. Students' perceptions of the interactive lecture

Students' responses to the four questionnaire statements are summarized in Table 3. Ratings were given on a five-point scale, with higher values indicating stronger agreement.

Table 3. Questionnaire responses in the interactive-lecture group ($n = 24$)

Statement	Mean	SD	Ratings of 4 or 5 (n)	Ratings of 4 or 5 (%)
The visualisations supported my understanding.	4.08	0.78	18/24	75.0%
The lecture maintained my attention.	4.04	0.69	19/24	79.2%
I understood the relationship between the different representations.	3.83	0.87	15/24	62.5%
I felt actively involved.	3.96	0.62	19/24	79.2%

The responses were generally favourable. The statement about relationships among representations received the lowest mean rating and the widest spread of responses. Thus, students reported attention and participation more consistently than understanding of the connections that formed the main instructional focus. These self-reports describe students' experience of the lecture; they do not provide an independent measure of achievement.

5. Conclusion

This study compared a traditional lecture and an interactive lecture on complex numbers among first-year engineering students. The groups had close reported mean scores on the prerequisite diagnostic test, although this descriptive comparison does not establish equivalence between them. On the post-test administered immediately after instruction, the interactive group achieved a higher mean total score (5.92 versus 4.12 out of 10; $p = 0.035$, Hedges' $g = 0.66$).

The interactive group also obtained a higher mean score on the six conversion and coordination questions (3.38 versus 2.47 out of 6), although this difference was not statistically significant at the 0.05 level ($p = 0.117$). Higher performance was also observed on the three within-representation processing questions (2.04 versus 1.35 out of 3). The findings therefore do not establish an advantage specific to conversion and coordination. Questionnaire responses indicated that most students felt attentive and actively involved, while agreement was less consistent concerning their understanding of relationships among representations.

The results suggest that a short interactive lecture may be a useful option for introducing complex numbers. However, the study involved a small sample of 41 students from two existing groups at one university. Non-random group assignment and assessment immediately after a single lecture limit causal conclusions and provide no evidence of lasting differences. Further studies with larger samples, random assignment where feasible, and

delayed post-tests are needed to examine the persistence of the observed differences and students' development of representational coordination.

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