Original Contribution

RESEARCH ON THE LIKELIHOOD RATIO OF REGISTERED UNIDENTIFIED SIGNALS OF SATELLITE ENVIRONMENTAL MONITORING OF ATMOSPHERE

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Abstract: Optical studies from the board of satellites are among the most informative. From space, in certain narrow spectral windows of the atmosphere environmental monitoring of the atmosphere is possible to be implemented. Objects of study are natural sources of environmental pollution and anthropogenic sources of pollution.

Key words, atmosphere environmental monitoring

The research of signals from satellites at different background brightness is one of the basic problems which concerns a lot of scientists who work in the sphere of atmospheric monitoring, space physics and distant methods [2,3,6,7,10], in the visible and near infrared part of the optic specter [1,4,5,8,9].

If there is no a priori information about the presence of a signal and this is a typical case in the research process, the likelihood ratio is the only thing that can be concluded from the research. Sometimes in such cases we have 50 % probability: absence or presence of a signal, i.e.

$$P(S)=P(O),$$

where P(S) – a posteriori probability for presence of a signal;

P(O) – a posteriori probability for absence of a signal;

It turns out that the likelihood ratio absolutely characterizes the probability for the presence of a signal in the realization. For most of the observational systems, a very small a priori likelihood for the presence of a signal is characteristic, i.e. *P(S) << 1*, and the a posteriori likelihood becomes a proportion from the likelihood ratio

To solve practical problems, the likelihood ratio can be presented by distributing the density of the realizations probability. First the realization of absence of signal is looked at *P(Y/O),* i.e. when the realization *y(t)* is only noises.

When the realization is known for a certain period of time τ, with intervals *H*, and at the end of every interval (in points $t_1, t_2 .., t_i .., t_H$) are

recorded the reports $y_i = y(t_i)$, which correspond in the given situation the values of the noise at moments of the time t_i. The sum of reports for the time τ make the kind of Y which can be presented as H-dimensional vector which make y_i, where $i = 1, 2, \ldots, H$, and the value of the noise in every point t_i is within the limits from y_i to $y_i + d_{yi}$, i.e.:

$$(1) \qquad y_1 < \text{n}_i < y_1 + d_{yi}\,.$$

If we use what we know from the probabilities theory, the relationship between the probability P and the probability density $\omega(x)$ of a random value X with probable definitions x, so:

$$(2) \qquad \omega(x) = \lim_{\Delta x \to 0} \frac{P(x < X < x + \Delta x)}{\Delta x},$$

and with multiple distribution:

$$(3) \qquad \omega(x_1, x_2, \ldots, x_n) = \lim_{\Delta x_1, \ldots, \Delta x_n \to 0} \frac{x_1 < X_1 < x_1 + \Delta x_1, \ldots, x_n < X_n < x_n + \Delta x_n}{\Delta x_1 \Delta x_2 \ldots \Delta x_n}\,.$$

On the basis of (3) we get:

$$(4) \quad P(Y/O) = \omega_n(y_1, y_2, \ldots, y_H)\,dy_1 dy_2 \ldots dy_H\,.$$

The last formulae can be presented in short

$$(5) \qquad P(Y/O) = \omega_N(Y)\,dY\,,$$

where: $dY = dy_1 dy_2 \ldots dy_H$ - element of the volume from H − dimensional space
$\qquad \omega_N(Y) \qquad H$-dimensional distribution of the noise probability.

Analogically we get the likelihood of the element Y to be present in the volume dY in a situation of signal presence:

$$(6) \qquad P(Y/S) = \omega(Y/S)\,dY\,,$$

where $\omega(Y/S)$ - relative probability density for realization of Y in a mixture of signal and noise.

Having in mind formulas (5) and (6), the relationship of the relative probability for realization is defined as likelihood ratios, we present it as A_p and the result is:

$$(7) \qquad A_p = \frac{\omega(Y/S)}{\omega_N(Y)}\,.$$

Very often the mixing of signal and noise is an algebraic sum, i.e. it is

signal and additive noise. We often have this case in the experiments:

$$(8) \qquad y_i = n_i + S_i,$$

where n_i - value of the noise at a moment of time t_i

S_i - value of the signal at the same moment of time.

In this case the probability for realization of the quantity y_i coincide with the probability for the noise realization with the quantities

$$n_i = y_i - S_i.$$

This means that the probability for realization of the ordinate y_i coincides with the probability to have the ordinate

And these parameters are not absolutely known, the signal form is also unknown and only the realization Y is known. We can presume that the parameters of the signal have random character a_k and the probability density is also known ω_A of their quantities at a certain moment of time:

$$(12) \qquad \omega_A(a_1, a_2, ..., a_k) \ .$$

Then the relationship of the likelihood ratio $P(Y/O)$ from the signal does not depend on $\omega_N(Y)dY$, but the probability $P(Y/S)$ at a certain realization at the presence of a random probable signal is defined by

$$y_i - S_i,$$

In a realization which has only noise. Consequently, when we have additivity ff signal and noise, the result is:

$$(9) \qquad \omega(Y/S) = \omega_N(Y - S),$$

and:

$$(10) \qquad A_p = \frac{\omega_N(Y - S)}{\omega_N(Y)}.$$

A case is researched for a likelihood ratio with unknown parameters when the signal depends on certain $A_1, A_2, ..., A_k$ and the time and it can be presented like:

$$(11) \qquad s(t, A_1, A_2, ..., A_k),$$

every possible value of the parameters..

To define *P(Y/S)* we use the formula of the probability density for a certain event *B* which happens with the likelihood $P(B/A_j)$ at the condition of the event occurrence A_j, which happens only with a certain probability $P(A_j)$:

$$(13) \qquad P(B) = \sum_j P(A_j)P(B/A_j),$$

and $\sum_j P(A_j) = 1$.

And in the case of the event A_j, all parameters of the signal are in the set system in the interval

(14) $a_{j1} \le A_1 < a_{j1} + \Delta a_{j1},......, a_{jk} \le A_k < a_{jk} + \Delta a_{jk}$,

and the full set of indexes $(j_1, j_2,..., j_k)$ is necessary to ensure the full coverage of the k-dimensional area of the possible values of the parameters $A_1, A_2,..., A_k$. Because of this, the

probability $P(A_j)$ is presented by the density of the parameters distribution $\omega_A(a_1, a_2,..., a_k)$:

$$(15) \quad P(A_j) = \omega_{Aj}(a_{j1}, a_{j2},..., a_{jk}).\Delta a_j, \Delta a_{j2}...a_{jk} .$$

The conditional probability $P(B/A_j)$ is by itself a probability to get the realization Y in the element of the volume dY on the condition that the parameters of the signal are within a

certain interval and they have certain values

$a_1 = a_{j1}, a_2 = a_{j2},..., a_k = a_{jk}$:

$$(16) \quad P(B/A_j) = P(Y / a_{j1}, a_{j2},..., a_{jk}) = \omega(Y / a_{j1}, a_{j2},..., a_{jk})dY .$$

When we substitute (15) and (16) into (13), it can be observed that the sum in (13) is of integral type and the

when the intervals $\Delta a_{j1}, \Delta a_{j2}...\Delta a_{jk}$ have a tendency towards zero, we get a k-multiple integral:

$$(17) \quad P(Y / S) = dY \iint_k \omega(Y / a_1, a_2,...., a_k)\omega_A(a_1, a_2,...., a_k)da_1 da_2,...., da_k ,$$

And the likelihood ratio becomes:

$$(18) \quad A_p = \frac{\iint_k \omega(Y / a_1, a_2,...., a_k)\omega_A(a_1, a_2,...., a_k)da_1 da_2,...., da_k}{\omega_N(Y)} .$$

If the relationship is represented:

$$(19) \quad \frac{\omega(Y / a_1, a_2,...., a_k)}{\omega_N(Y)} = A_p(a_1, a_2,..., a_k) ,$$

the final formula for the likelihood ratio is:

$$(20) \quad A_p = \iint_k A_p(a_1, a_2,...., a_k)\omega_A(a_1, a_2,...., a_k)da_1 da_2,...., da_k ,$$

where $A_p(a_1, a_2,..., a_k)$ characterize the probability of presence of a signal with certain parameters $(a_1, a_2,..., a_k)$.

It can be concluded that with random parameters $(A_1, A_2,..., A_k)$, which have probability density $\omega_{Aj}(a_j)$

their integration with all of their values gives the mathematical expectancy or the average value of the likelihood ratio. Formula (20) gives the probability of the signal presence at different possible values of its random parameters.

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