

Journal scientific and applied research, vol. 4, 2013 Association Scientific and Applied Research International Journal

Original Contribution

ISSN 1314-6289

A SURVEY OF PHASE MANIPULATED SIGNALS WITH HIGH STRUCTURAL COMPLEXITY AND SMALL LOSES AFTER PROCESSING WITH MISMATCHED FILTERS

Tsvetoslav Tsankov, Tihomir Trifonov, Lilia Staneva

KONSTANTIN PRESLAVSKI UNIVERSITY OF SHUMEN, SHUMEN 9712, 115, UNIVERSITETSKA STR.

e-mail: hitar@abv.bg, trif.69@abv.bg, anest_bg@bitex.bg

Abstract: In the paper the results of a computer survey of the uniform phase manipulated signals with great structural complexity and small lose in the signal-to-noise ratio after processing with mismatched filters are presented. These results can be used in the development of radars with high resistance to hostile radioelectronic environment and ability for discovering and resolution of small objects.

Key words: *synthesis of signals for radars, phase manipulated signals with high structural complexity.*

1. **Introduction**

Today the radar systems must satisfy a large number of technical requirements. The most important of them are: ability for discovering and resolution of small objects and high resistance to the radio-electronic counter-measurements [1], [2]. The general method for providing of antijamming capabilities is the usage of pseudo-noise signals with high structural complexity (SC), which are hard both to detect and to imitate [3] - [11]. Unfortunately, the autocorrelation functions (ACF) of these signals have high side-lobes and as a result the echo-signals of large targets mask the echo-signals of small targets. This contradiction can be solved by processing of echo-signals with mismatched filters (MMFs),

which eliminate the side-lobes of the ACFs. The cost of this approach is the diminishing of the signal-to-noise ratio (SNR) [1], [2].

With regard the results of a computer survey of the uniform phase manipulated (PM) signals with great SC and small lose in the SNR after processing with MMFs are presented in our paper.

The paper is organized as follows. The methodology and restrictions of the survey are given in Section 2. In Section 3 the results of the survey are presented. Conclusions of the paper are summarized in Section 4.

2. Methodology and restrictions of the survey

The survey is based on the fact that any PM signal can be presented as a sequence of complex numbers [1]-[11]

(1)
$$
\{\zeta(i)\}_{i=0}^{N-1} = \{\zeta(0), \zeta(1), ..., \zeta(N-1)\}
$$

called *signal sequence* or simply *PM signal*.

In (1) the length *N* denotes the quantity of the consecutive elementary phase pulses (chips), forming the PM signal.

The complex number

(2)
$$
\zeta(i) = U_{mi} e^{j\psi_i}, j = \sqrt{-1}
$$

is the so-named *complex envelope* of the *i*-th chip. It presents the amplitude U_{mi} and the phase angle ψ_i of the *i*th chip.

Today the so-named *uniform PM signals*, which satisfy simultaneously the following conditions

(3)
$$
U_{mi} = U_{m0} = const, i = 1 \div N - 1
$$

$$
(4) \qquad \psi_i \in \left\{ \frac{2\pi}{p}l, \ l = 0 \div p - 1 \right\}
$$

are preferred due to the following reasons [1]-[11]. First, the condition (3) minimizes the probability of detection of the PM signal by the enemy radio-electronic intelligence as the signal spectrum is uniformly distributed. Second, the observation of the condition (4) leads to simplification and reduction the cost of the communication devices.

In case of uniform PM signals the complex envelopes of the chips become the form [1]-[11]

(5)
$$
\zeta(i) = U_{m0}.e^{j\frac{2\pi}{p}s(i)}
$$

where the integer sequence

(6)
$$
S = \{s(0), s(1), \ldots, s(N-1)\},
$$

$$
s(i) \in \{0, 1, \ldots, p-1\} = Z_p
$$

is called *the power sequence of the uniform PM signal* or simply *the power sequence*.

With regard to the positive features of the uniform PM signals they are studied intensively during the past several decades $[1]$ – $[11]$. Among these signals the class of the so-named *bent-function sequences* has gained the most attention, because the bent functions have a very high SC, providing immunity of the communication systems to the radioelectronic counter-measurements.

The bent-function sequences are families of uniform PM signals, which power sequences are generated by the rule [1] - [9], [11]:

$$
s_j(i) =
$$

(7) = $f\left[tr_1^n(\beta_0 \alpha^i), t r_1^n(\beta_{m-1} \alpha^i)\right] +$
+ $j^T \cdot \vec{X} + tr_1^n(\eta \cdot \alpha^i)$

In (7) the following notations are used:

1) $s_j(i), j = 0 \div K - 1,$ $i = 0 \div N - 1$ is the *i*-th element of the

j-th power sequence of the family of uniform PM signals;

2) $N = p^n - 1$ is the length of the uniform PM signals from the family, *р* is prime, and *n*, *m* and *k* are positive integers, connected with the relation:

(8)
$$
n = \begin{cases} 2m = 4k, & \text{if } p = 2; \\ 2m, & \text{if } p \neq 2; \end{cases}
$$

3) *K* is the quantity of the uniform PM signals in the family $(K = p^m = p^{n/2});$

4) α is a primitive element of the finite algebraic field $GF(p^n)$;

5) $\beta_0, \beta_1, ..., \beta_{m-1}, ...$ are a basis of $GF(p^m)$ over $GF(p)$; it is convenient to use

(9)
$$
\beta_0 = 1, \beta_1 = \beta^1, ..., \beta_{m-1} = \beta^{m-1}
$$
,

where

$$
(10) \qquad \beta = \alpha^d, d = p^m + 1
$$

is a primitive element of the finite algebraic field $GF(p^m)$ [12];

6) the parameter η is chosen according to the condition

$$
(11) \quad \eta \in GF(p^n)/GF(p^m);
$$

it is convenient to take $\eta = \alpha$;

7) the vector inner product $\vec{j}^T \vec{X}$ $(\cdot, T"$ means matrix transposition) determines the number of the uniform PM signal in the family;

8) $f(x_0, x_1, \ldots, x_{m-1})$ is a bentfunction, mapping the elements of $GF(p^m)$ in the elements of $GF(p)$;

9) $tr_1^n(z)$ is the trace-function, mapping the elements *z* of $GF(p^n)$ in the elements of $GF(p)$ [12]:

(12)
$$
tr_1^n(z) = z^{p^0} + z^{p^1} + \dots + z^{p^{n-1}}
$$
.

On the base of the family of power sequences, generated by the formulae (7), the chips of the uniform PM signals from the family are formed according to the rule

$$
(13) \t u_j(i) = w^{s_j(i)}.
$$

Here $u_j(i)$, $j = 0, 1, ..., K-1$, $i = 0, 1, \dots, N-1$ is the complex envelope of the *i*-th elementary pulse (chip) of the *j*-th uniform PM signal from the family and *w* is *р*-th root of the unity.

The survey was conducted under the following restrictions.

First, the length of the uniform PM signals was in the range

(14)
$$
N = 4 \div 10000.
$$

This limitation was inspired by the fact that the complexity of communication devices grows rapidly when the signal length increases.

Second, the classes of the conventional and extended binary and ternary bent-function sequences were explored. These restrictions can be explained as follows.

At the one hand, the binary and ternary bent-function sequences are the most convenient from the point of view of the practical implementation, because they are generated by the simplest types of the phase manipulation – the binary and ternary phase shift-keying.

At the other hand, the conventional binary bent-function sequences exist only for the signal lengths

(15)
$$
N = 2^{4k} - 1, k = 1, 2, 3,
$$

As a result in the range (14) only the signal lengths

$$
(16) \quad N=15,255,4095.
$$

are possible.

In order to obtain a more dense set of signal lengths, the survey was conducted in the classes of the conventional and extended binary and ternary bent-function sequences.

The class of extended bentfunction sequences is similar to the above described class of the conventional bent-function sequences. Anyway the following distinctions are important.

First of all, the parameter *m* is simply chosen to be a factor of *n*, i.e.

$$
(17) \qquad \qquad n=m.l, l>1.
$$

As a result, (10) must be changed to

(18)
$$
\beta = \alpha^d,
$$

\n
$$
d = p^{(l-1)m} + p^{(l-2)m} + ... + 1
$$

Second, it is allowed the function f in (7) to be any non-linear function, mapping the elements of $GF(p^m)$ in the elements of $GF(p)$.

Third, the restriction (11) is not observed and it is allowed the parameter η to be any element of $GF(p^n)$.

3. A survey of phase manipulated signals with high structural complexity and small loses after processing with mismatched filters

The survey of phase manipulated signals with high structural complexity was conducted according to the methodology and restrictions, presented in the previous section. The main objective of the survey was finding of uniform PM signals with high SC and small loses in SNR after processing with MMFs. As known, these loses are measured by the *coefficient of loses* (KS) γ [1], [2]:

(19)
$$
\gamma = \frac{Q_{MF}^2}{Q_{MMF}^2} = \sum_{i=0}^{N-1} \frac{1}{|C_i|^2}.
$$

Here Q_{MF}^2 is the SNR in the output of the receiver after processing the PM signal with the corresponding matched filter (MF). In the presence of additive Gaussian noise with zero mathematical expectation and dispersion σ^2 , Q_{MF}^2 is given by

$$
(20) \hspace{1cm} Q_{MF}^2 = \frac{N}{\sigma^2}.
$$

Analogously, Q_{MMF}^2 is the SNR in the output of the receiver after processing the PM signal with the corresponding mismatched filter (MMF), which eliminates the sidelodes of the periodic ACF (PACF):

(21)
$$
Q_{MMF}^2 = \frac{N}{\sigma^2} \left(\sum_{i=0}^{N-1} \frac{1}{|C_i|^2} \right)^{-1}
$$
.

Here and in (19) $\{C_0, C_1, \dots, C_{N-1}\}$ is the spectral sequence, corresponding to the PM signal $\{\zeta(i)\}_{i=0}^{N-1}$ $=$ *N* $\left\langle \zeta(i) \right\rangle_{i=0}^{N-1}$. It is obtained by the Fourier"s transformation:

(22)
$$
C_k = \sum_{i=0}^{N-1} \zeta(i) e^{-j\frac{2\pi}{N}k \cdot i},
$$

$$
k = 0, 1, ..., N-1
$$

During the survey the algorithm for modeling of uniform PM signals, presented in our previous paper [13], was intensively used. This will be clarified by the most simple example, when $n = 2m = 4k = 4$ and the object of the researches is the class of binary bent-function sequences (i.e. $p = 2$).

At the beginning, by the algorithm from [13], a binary linear recurring sequence (LRS), representing the term $tr_1^4(\beta_0\alpha^i)$ 1 $tr_1^4(\beta_0 \alpha^i)$ in (7), is formed. As $\beta_0 = \beta^0 = 1$

according to (9), for the generating of $(\beta_0 \alpha^i) = tr_1^4(\alpha^i)$ $0^{(1)} = r_1$ 4 1 $tr_1^4(\beta_0\alpha^i) = tr_1^4(\alpha^i)$ any primitive

polynomial over $GF(2)$ with degree 4 can be used (for example $g(y) = y^4 + y^3 + 1$) [14]. From this LRS, the LRSs $tr_1^4(\beta_1\alpha^i)$ 1 $tr_1^4(\beta_1\alpha^i)$ and $^4(\eta\alpha^i)$ 1 $tr_1^4(\eta \alpha^i)$ are obtained by 5 cyclic shifts to the left (we recall that $\beta_1 = \beta^1 = \alpha^5$ according to (10)) and by *s* cyclic shifts to the left (during the survey the cases $\eta = \alpha^s$, $s = 0, 1, \dots, 14$ were studied).

After that, the bent-function *f* in (7) was chosen to be [15]

(23)
$$
f[tr_1^4(\beta_0 \alpha^i), tr_1^4(\beta_1 \alpha^i)] =
$$

$$
= tr_1^4(\beta_0 \alpha^i) tr_1^4(\beta_1 \alpha^i),
$$

$$
\beta_0 = \beta^0 = 1, \beta_1 = \beta^1 = \alpha^5.
$$

At the end of the example, it should be seen that the vector inner should be seen can be only

(24)

$$
\vec{j}^T \vec{X} = \begin{cases}\n0 \cdot tr_1^4(\beta_1 \alpha^i) + 0 \cdot tr_1^4(\beta_0 \alpha^i); \\
0 \cdot tr_1^4(\beta_1 \alpha^i) + 1 \cdot tr_1^4(\beta_0 \alpha^i); \\
1 \cdot tr_1^4(\beta_1 \alpha^i) + 0 \cdot tr_1^4(\beta_0 \alpha^i); \\
1 \cdot tr_1^4(\beta_1 \alpha^i) + 1 \cdot tr_1^4(\beta_0 \alpha^i); \\
\beta_0 = \beta^0 = 1, \beta_1 = \beta^1 = \alpha^5.\n\end{cases}
$$

Some of the most interesting results of the survey are presented in Table I.

TABLE I

The most interesting results of the computer survey of the uniform PM signals with great structural complexity and small lose in the SNR after processing with mismatched filters

4. Conclusion

In the paper the results of a computer survey of the uniform PM signals with great SC and small lose in the SNR after processing with MMFs are presented. These results prove that it is possible to provide simultaneously a high ability for discovering and resolution of small objects and a high resistance to the

References:

[1] V. P. Ipatov, *Periodical discrete signals with optimal correlation properties*. Moscow – Radio and Communication, 1992, 152 pp. (in Russian)

radio-electronic counter measurements in the radars.

The results of the survey can be successfully used in the process of development of perspective radar system, possessing high information capabilities and high reliability in hostile radio-electronic environment.

[2] V. P. Ipatov, *Spread spectrum and CDMA. Principles and Applications*. University of Turku and Saint Petersburg Electrotechnical University "LETI", 2006, 373 pp. (in Russian)

[3] E. L. Key, "An analysis of the structure and complexity of nonlinear binary sequence generators," *IEEE Trans. Inform. Theory*, vol. IT-22, pp. 732-736, Nov. 1976.

[4] J. D. Olsen, R. A. Scholtz and L. R. Welch, "Bent-function sequences," *IEEE Trans. Infirm. Theory*, vol. IT-28, pp. 858-864, Nov. 1982.

[5] J.-S. No and P. V. Kumar, "A new family of binary pseudorandom sequences having optimal periodic correlation properties and large linear span," *IEEE Trans. Inf. Theory*, vol. 35, no. 2, pp. 371–379, Mar. 1989.

[6] J.-W. Jang, Y.-S. Kim, J.-S. No, and T. Helleseth, "New family of pary sequences with optimal correlation property and large linear span," *IEEE Trans. Inf. Theory*, vol. 50, no. 8, pp. 1839–1844, Aug. 2004.

[7] P. V. Kumar and O. Moreno, "Prime-phase sequences with periodic correlation properties better than binary sequences," *IEEE Trans. Inf. Theory*, vol. 37, no. 3, pp. 603– 616, May 1991.

[8] S. Golomb, G. Gong, *Signal design for good correlation for wireless communications,*

cryptography and radar. Cambridge University Press, 2005, 455 pp.

[9] F. Chen, J. Hua, C. Zhau and S. Shou, "Fast generation of bent sequence family," *Inform. Technology J.*, 9, 2010, pp. 1397 – 1402

[10] L. Tong, J. Hua, L. Meng and S. Shou, "Correlation analysis and realization of Gordon-Mills-Welch sequences in advanced system," *Inform. Technology J.*, 10, 2011, pp. $908 - 913$

[11] S. S. Yudachev, "Sequences on the base of bent-functions for wideband systems with code-division of channels", *Engineers' gazette*, №1, Jan. 2013, pp. $1 - 11$ (in Russian)

[12] R. Lidl and H. Niederreiter, *Finite Fields, vol. 20, Encyclopedia of Mathematics and its Applications.* Amsterdam: Addison-Wesley, 1983.

[13] T. S. Tsankov, T. S. Trifonov, L. A. Staneva, An algorithm for synthesis of phase manipulated signals with high structural complexity, *J. Scientific and Appl. Research*, 2013 (accepted for publishing)

[14] N. Zierler, Linear recurring sequences, *J. Soc. Ind. Appl. Math.*, $7(1959)$, N^o₁, pp. 31 – 48

[15] O. S. Rothaus, "On 'bent' functions," *J. Comb. Theory*, Series A20, pp. 300-305, 1976.