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SOME PROBLEMS OF OPTIMAL MANAGEMENT OF ECONOMIC SYSTEMS

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Abstract. *The task of optimal management of economic system is examined. Sulution of the problem comes true through the method of the Pontryagin maximum principle. As a function of the equation of state will use the Gross Domestic Product (GDP).*

An optimal management of state of financial system United Kingdom from initial position to the set end position is offered.

Keywords: *investment, optimal management, GDP.*

JEL classification codes: *C01; C53; E17; F21*

Introduction

Practically any economic-financial system changes during time. For a management of the system it is necessary to have a management function. Changing the parameters of this function, it is possible to get motion of the system on an optimal trajectory. One method of solving the management problem is the method of the Pontryagin maximum principle [1,2,3,4]. As a function of the equation of state will use the Gross Domestic Product.

In order to find out required correlations it is necessary to take into account dependences between all parameters of the system. The process of finding the optimal functional relationship between the variables can be found in [5,6].

Problem statement

As an example, consider the problem of optimal management with the initial data for United Kingdom [7] on the interval $[t_0;T]$:

Table 1: Initial data for calculation

t	Year	GDP (Current Prices, US$ Billion), *Y(t)*	Investment (% of GDP), $\alpha_1(t)$	General government total expenditure (% of GDP), $\alpha_2(t)$	General government revenue (% of GDP), $\alpha_3(t)$
1	2011	2471,88	15,584	45,861	37,301
2	2012	2602,49	16,411	44,48	37,542
3	2013	2743,35	17,358	42,793	37,776
4	2014	2890,99	18,18	41,26	37,843
5	2015	3050,52	18,913	40,072	37,779
6	2016	3220,42	19,601	38,704	37,384

Processing the data in Table 1, we find the optimal function in the form $Y = ab^t$. Applying the method of the least squares, we obtain the unknown parameters: $a = 2342{,}0198$; $b = 1{,}053276$. This optimal function corresponds to the following boundary value problem:

$$\frac{dY(t)}{dt} = \ln bY(t)\ ;\ Y(t_0) = ab \tag{1}$$

Equation (1) is the equation of studied system state, with the initial condition.

Find the optimal functional dependence for $\alpha_2(t)$- general government total expenditure (% of GDP) on the interval $t \in [t_0; T]$:

$$\alpha_2(t) = 47{,}2492 - 1{,}444057t \tag{2}$$

Thus, we obtain a mathematical formulation of the overall cost management problem for the state

Problem A

The equation of state of the system:

$$\frac{dY(t)}{dt} = \ln bY(t) + U(t) \tag{3}$$

Initial conditions:

$$Y(t_0) = ab \tag{4}$$

Management function has following form:

$$U(t) = \alpha_2(t)ab^t \qquad (5)$$

where $\alpha_2(t)$ has the form (2).

The condition of optimality is set in form:

$$\int_{t_0}^{T} \exp(-\delta t)(\alpha_1(t) + \alpha_3(t) - \alpha_2(t))Y(t)dt \to \max \qquad (6)$$

where δ - discount coefficient; $\alpha_1(t), \alpha_3(t) > 0$.

Taking into account data from tables 1 we will get:

$$\alpha_1(t) = 0{,}148332 + 0{,}008118t \text{ ; } \alpha_3(t) = 0{,}376 \qquad (7)$$

Problem statement.

It is necessary to find the optimal value of parameter $\alpha_2(t)$, on which management of state of the financial system is executed. A size reflecting financial receivabless and charges (6) is maximized.

To solve the problem we apply the Pontryagin maximum method [3].

We write the Hamiltonian function:

$$H(t) = \Psi(t)\{\alpha_2(t)ab^t + \ln bY(t)\} + \exp(-\delta t)\{\alpha_1(t) + \alpha_3(t) - \alpha_2(t)\}Y(t) \qquad (8)$$

where $\Psi(t)$ - auxiliary function that satisfy equation

$$\frac{d\Psi(t)}{dt} = -\Psi(t)\ln b - \exp(-\delta t)\{\alpha_1(t) + \alpha_3(t) - \alpha_2(t)\} \qquad (9)$$

We will find the extremum of Hamiltonian function on the parameter of management :

$$\frac{dH}{d\alpha_2} = -Y(t)\exp(-\delta t) + \Psi(t)ab^t \qquad (10)$$

Using the condition of extremum, we will get:

$$\Psi(t) = \frac{\exp(-\delta t)Y(t)}{ab^t} \qquad (11)$$

Using (11) and taking (3) into account we will find:

$$\frac{d\Psi(t)}{dt} = \exp(-\delta t)\alpha_2(t) - \frac{\delta \exp(-\delta t)Y(t)}{ab^t} \qquad (12)$$

We put (11) and (12) in (9) :

$$Y(t) = -\frac{ab^t(\alpha_1(t) + \alpha_3(t))}{\ln b - \delta} \qquad (13)$$

Set:

$$\alpha_4(t) = \alpha_1(t) + \alpha_2(t) \qquad (14)$$

Putting (13) in equalization of the state (3), we will get a management function in a form:

$$\alpha_2(t) = -\frac{1}{\ln b - \delta}\frac{d\alpha_4(t)}{dt} \qquad (15)$$

Thus, we get a next boarding problem that corresponds to the optimal function of management :

The equation of state of the system

$$\frac{dY(t)}{dt} = \ln bY(t) - \frac{\alpha_4'(t)}{\ln b - \delta}ab^t \qquad (16)$$

Initial conditions:

$$Y(t_0) = ab \qquad (17)$$

Assuming $\alpha_4'(t) = const$, than solution of task (16), (17) has following form :

$$Y(t) = ab^t\{1 - \alpha_4'(t) + \alpha_4'(t)t\} \qquad (18)$$

Numerical examples

We will enter next basic data

For founding parameter $\alpha_4(t)$ we use correlations (7). Putting initial data in (15), we will get the value of optimal managing function $\alpha_2 = 0{,}1685$. Thus, if during investigated period a managing function will be equal to the got value, then an optimization function (6) will take on a maximal value taking into account discounting. A formula (18) allows to define the function of the system state that corresponds to the optimal function of management.

Problem B

Statement of problem A is examined. Additionally to the terms (3) - (6) we will add additional limitations:

1. condition of final system in the set eventual state

$$Y(T) = Y_T \tag{19}$$

2. limit on the function of management

$$\beta_1 \le \alpha_2(t) \le \beta_2 \tag{20}$$

We will find the solution of equalization of the state (3) with an account (20). Solution of equalization, satisfying correlation (4) has form :

$$Y(t) = ab^t\{1 - \beta_1 + \beta_1 t\} \tag{21}$$

$$Y(t) = ab^t\{1 - \beta_2 + \beta_2 t\} \tag{22}$$

Satisfying (19), we will found solution in a form:

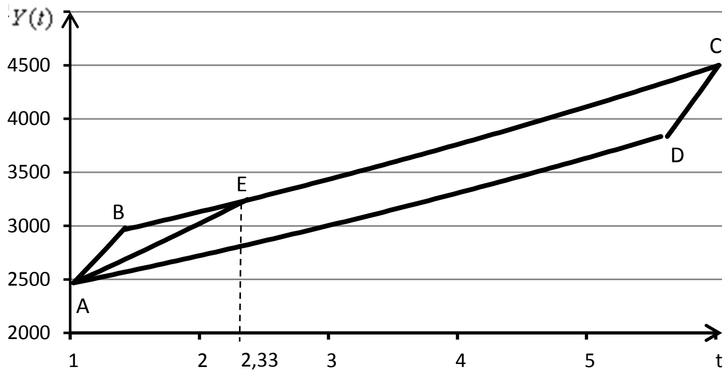
$$Y(t) = (Y_T - \beta_1 Tab^T)b^{(t-T)} + \beta_1 tab^t \tag{23}$$

$$Y(t) = (Y_T - \beta_2 Tab^T)b^{(t-T)} + \beta_2 tab^t \tag{24}$$

Building on a function plane set by correlations (18), (21) - (24), we will get an optimal management of the system state from initial position (4), in eventual (19). Thus motion has to be done on an optimal trajectory (18), but it must not go beyond the borders of possible solutions. Depending on the initial and eventual state, it is possible to make time switching from one trajectory to other. Thus an optimal function (6) will reach its maximum.

Numerical examples

As basic data we will take data from data A, and also additional data in a form $Y(6) = 4500; \beta_1 = 0,05; \beta_2 = 0,45$. On a picture 1 an optimal management of system state from the initial state in the set eventual state is presented. The area of acceptable values of state of investigated system is set as ABCD. The optimal transition of the system done firn initial state (point A) on an optimal trajectory (6) to the point E. From a transit point (point E) to the eventual state (point C) the system moves on trajectory(23). Switching time from one trajectory to other, for the set initial conditions takes place at $t_E = 2,33$.



Picture 1 Optimal management GDP, US$ Billion

Conclusion

A decision over of a few tasks of optimal management is brought through the method of the Pontryagin maximum principle. An optimal management of system state is offered from initial position to the set eventual position. It is shown that an offered algorithm works and has practical application. Numeral results of the financial indexes of United Kingdom for 2011-2016 are shown.

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