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ANALYSIS OF THE CONDITIONS FOR SYNTHESIS OF EFFICIENT SIDE-LOBES SUPPRESSION FILTERS FOR PHASE MANIPULATED SIGNALS

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Abstract: *The radio signals, possessing the so-named ideal auto – correlation function, resembling the Dirac delta – function, have a critical role for the wireless communication systems, especially for radars, radio-navigation systems, radio – synchronization and so on. Despite of this at the moment only few classes of such signals are invented. With regard in the paper we analyze in more detail the method for synthesis of pairs of phase manipulated signals and mismatched side-lobe suppression filters (SLSFs), which modify the real periodic auto – correlation function of the signals in the so-named ideal form, resembling the Dirac delta-function. As a result a new necessary condition for synthesis of efficient SLSFs is proven.*

Key words: *digital signal processing; signals with ideal periodic auto-correlation; side-lobe suppression filter.*

1. Introduction

The radio signals, possessing the so-named ideal *auto – correlation function* (ACF), resembling the Dirac delta – function (pulse), have a critical role for the wireless communication systems, especially for radars, radio-navigation systems, radio – synchronization and so on. Due to this reason great efforts have been directed to finding of radio signals with ideal ACFs. Despite of this at the moment only few classes of such signals are invented [1], [2], [3].

With regard to above described situation, in the paper we analyze in more detail the method for synthesis of pairs of periodic *phase manipulated* (PM) signals and mismatched *side-lobe suppression filters* (SLSFs) [2], which modify the real periodic ACF (PACF) of the signals in an ideal PACF. As a result a new necessary condition for synthesis of efficient SLSFs is proven.

The paper is organized as follows. First, the factors, influencing on the coefficients of losses in *the signal-to-noise ratios* (SNRs) of the SLSFs, are analyzed in Section 2. On this base a new condition, which diminishes the coefficients of losses of the SLSFs, is proven. After that the correctness of the new condition is demonstrated for the case of periodic PM signals, which are repetitions of Barker signals [4]. At the end, the conclusions of the paper are summarized in Section 3.

2. Analysis of the conditions for synthesis of efficient side-lobes suppression filters for phase manipulated signals

As known, the signal in the output of the receiver of a radio - communication system can be presented in the form [4], [5], [6]

(1)
$$\begin{aligned} &Q_{uv}(x) = \\ &= \left(u_{N-1}x^{N-1} + \ldots + u_1 x + u_0\right) \times \\ &\times \left(v_{N-1}^* x^{-(N-1)} + \ldots + v_1^* x^{-1} + v_0^*\right) \\ &\bmod(x^N - 1) \end{aligned}$$

In (1) the following notations are used.

First, $Q_{uv}(x)$ is the generating function (polynomial) of *the periodic cross-correlation function* (PCCF) of the received PM signal and the impulse characteristic of the receiver

(2)
$$\begin{aligned} &Q_{uv}(x) = \\ &= q_{N-1}x^{N-1} + q_{N-2}x^{N-2} + \ldots + q_0 \\ &\bmod(x^N - 1) \end{aligned}$$

Here N is the quantity (the number) of the elementary phase impulses (chips), forming the PM signal.

Second, $\{u_0, u_1, \ldots, u_{N-1}\}$ are the digital samples of the received signal and, analogously, $\{v_0, v_1, \ldots, v_{N-1}\}$ are the digital samples of the impulse characteristic of the receiver.

Third, the symbol „*” means complex conjugation.

In order to maximize SNR the receiver is constructed as a filter, matched to the signal, used in the communication system, i.e.

(3)
$$v_0 = u_0, v_1 = u_1, \ldots, v_{N-1} = u_{N-1}$$

In this case the output signal of the receiver is presented by the polynomial $Q_{uu}(x)$, which coefficients are the samples of the PACF of the PM signal [4], [5], [6].

Unfortunately very often the employment of matched filters leads to PACFs with great side - lobes $\{q_1, q_2, \ldots, q_{N-1}\}$, which mask the main - lobes

(q_0) of weak signals, which may be more important. A general approach to avoid this negative effect is the usage of a mismatched filter, which impulse characteristic preserves the main-lobe (q_0) of the PACF, but removes all the side – lobes [2]. In other words, in (1) the samples $\{v_0, v_1, ..., v_{N-1}\}$ instead to satisfy condition (3) are chosen to provide PACF with ideal form

$$q_0 = N, \quad q_1 = q_2 = q_{N-1} = 0 \tag{4}$$

Such mismatched filters are named *side-lobe suppression filters* (SLSFs) [2].

The cost of this valuable result is a diminishing of the SNR. Due to this reason the development of methods for synthesis of PM signals, which can be processed by SLSFs with small losses in the SNR, have great practical importance [2], [5]. With regard to this conclusion, it is necessary to analyze the factors, which determine the losses in the SNR.

As known [2], the coefficient of losses in SNR, when a periodic PM signal is processed by the respective SLSF, is

$$\gamma = \sum_{i=0}^{N-1} \frac{1}{|C_i|^2}. \tag{5}$$

Here $\{C_0, C_1, ..., C_{N-1}\}$ is the digital Fourier's spectrum of the received signal $\{u_0, u_1, ..., u_{N-1}\}$

$$C_l = \sum_{k=0}^{N-1} u_k \left(e^{j\frac{2\pi}{N}l} \right)^k, . \quad l = 0,1,...,N-1 \tag{6}$$

It is not hard to show [2] that $\gamma = 1$ only if

$$|C_0| = |C_1| = ... = |C_{N-1}| = \sqrt{N}. \tag{7}$$

Unfortunately, today known PM signals, which satisfy (7), have lengths, restricted by the number *m* of the levels of the used phase manipulation [1], [2]

$$N \le 2m. \tag{8}$$

Consequently, if longer PM signals are needed more complex phase manipulation must be exploited.

Obviously, the restriction (8) can be avoided by application of PM signals, possessing a quasi-uniform (or nearly flat) spectrum

(9) $$|C_0| \approx |C_1| \approx \ldots \approx |C_{N-1}|.$$

Here it should be accounted that after substitution $x = e^{j\frac{2\pi}{N}l}$, $l = 0,1,\ldots,N-1$ in (1), the result is

(10) $$|C_l|^2 = \sum_{k=0}^{N-1} q_k \left(e^{j\frac{2\pi}{N}l} \right)^k, \quad l = 0,1,\ldots,N-1.$$

This is the digital analog of the well-known Wiener-Khinchin theorem [1], [2], [3].

From (10) it follows that in the case of the ideal PACF, described by (4), the spectrum of the PM signal is flat, satisfying condition (7). With regard to this conclusion it is naturally to explore the more general class of PM signals with the so-named *two levels PACF*

(11) $$q_0 = N, \quad q_1 = q_2 = q_{N-1} = Q_0$$

because it contains the PM signals with ideal PACF (i.e. the PM signals with $Q_0 = 0$) as a sub-class.

It is shown [2], that the coefficient of losses in the SNR, when a binary PM signal with two levels PACF is processed by the respective SLSF, is

(12) $$\gamma = \frac{SNR_{MF}}{SNR_{SLSF}} = \frac{1}{N + (N-1).Q_0} + \frac{N-1}{N - Q_0} \geq 1$$

Here SNR_{MF} is the SNR in the case of the matched filter, SNR_{SLSF} is the SNR in the case of the SLSF.

The analysis of (12) can be resumed as follows.

First, due to the physical limitations, the amplitude Q_0 of the side-lobes of the PACF of a binary PM signals with two levels PACF must be in the range

(13) $$-1-\frac{1}{N-1}\le Q_0\le N$$

Second, in the range (13) the coefficient of losses in the SNR possesses only one local minimum $\gamma=1$, located at the point $Q_0=0$. This can be explained by the fact that when $Q_0=0$ the PACF has ideal form and it is not necessary to replace the matched filter with a SLSF.

Third, the most frequently exploited in the practice binary PM signals with two levels PACF such as the *maximal length sequences* (m – sequences), the *Gordon-Mills-Welch* (GMW) *sequences*, the *Legendre* (or *quadratic residue*) *sequences*, the *Hall sextic residue sequences*, the *twin-prime sequences*, have PACF with $Q_0=-1$ [1], [2], [3]. For all these PM signals

(14) $$\lim_{N\to\infty}\gamma=2,$$

which is not appropriate in the most practical situations due to the loss of the half of the energy of the received signals.

Forth, if Q_0 is in the range

(15) $$+1\le Q_0\le\frac{N+1}{2}$$

then

(16) $$+1<\gamma\le 2.$$

From (15) and (16) it can be concluded that the efforts for developing of methods for synthesis of PM signals, which can be processed by SLSFs with small losses in the SNR, should be directed to finding binary PM signals with two levels PACF with the side-lobes in the range (15).

This very important conclusion will be clarified by binary PM signals, which are periodic repetition of the *Barker signals* (*codes*) [4], [6]. The discrete mathematical model of these PM signals is:

$$u[k] = \\ = \{\ldots, u_0, u_1, \ldots, u_{N-1}, u_0, u_1, \ldots, \\ u_{N-1}, \ldots, u_0, u_1, \ldots, u_{N-1}, u_0, \ldots\} \tag{17}$$

As known [3], the samples of one period of the Barker signals have the values, shown in Table I.

Table I
The digital samples of the Barker signals

N	$\{u_0, u_1, \ldots, u_{N-1}\}$
2	$\{1,-1\}$, $\{1,1\}$
3	$\{1,1,-1\}$
4	$\{1,1,-1,1\}$
5	$\{1,1,1,-1,1\}$
7	$\{1,1,1,-1,-1,1,-1\}$
11	$\{1,1,1,-1,-1,-1,1,-1,-1,1,-1\}$
13	$\{1,1,1,1,1,-1,-1,1,1,-1,1,-1,1\}$

As the Barker signals with even lengths are trivial ($N = 2$) or have ideal PACF ($N = 4$), the attention will be focused on the SLSFs of the Barker signals with odd lengths $N = 3, 5, 7, 11, 13$.

It is important to point out that the periodic repetition of a Barker signal possesses two levels PACF. More specifically [3]:

$$Q_0 = \begin{cases} +1, N = 4m+1; \\ -1, N = 4m+3. \end{cases} \tag{18}$$

The coefficients of losses in the SNR, when the periodic repetitions of the Barker signals are processed by the respective SLSFs, are presented in the Table II.

As seen, the coefficients of losses in SNR of the periodic repetitions of the Barker signals, which side-lobes are $Q_0 = -1$, tend rapidly to 2. In contrast in the case $Q_0 = +1$ the coefficients of losses tend to 1. These results are in a full accordance with (14) and (16). They can be explained by the fact that the energy spectrums of the periodic repetitions of the binary Barker signals with $Q_0 = +1$ are nearly flat

(19)

$$N = 5,\ |C_0|^2 = 3,$$
$$|C_1|^2 = |C_2|^2 = \ldots = |C_4|^2 = 2$$

(20)

$$N = 13,\ |C_0|^2 = 5,$$
$$|C_1|^2 = |C_2|^2 \ldots = |C_{12}|^2 = 3,4641$$

Table II
The coefficients of losses in the SNR of periodic repetitions of the binary Barker signals

N	Q_0	γ
3	-1	1,5
5	+1	1,11
7	-1	1,75
11	-1	1,8333
13	+1	1,04

Unfortunately, binary Barker signals with odd lengths exist only for lengths $N \le 13$ [2], [3]. With regard to this restriction the so-named *generalized Barker codes* (*signals*) have been developed [3]. These PM signals possess also ACFs with unit magnitude of the side-lobes, but they are generated by means of more complex phase manipulations ($m > 2$).

In order to verify the correctness of the above analysis, the SLSFs for the periodic repetitions of generalized Barker signals with lengths $N = 14,15,18$ have been synthesized and explored as follows

(21)

$$S_{14,6} = \{1,-\omega^2,-1,\omega^2,\omega^2,-1,$$
$$\omega^2,\omega,\omega^2,-\omega^2,-\omega,\omega^2,\omega,1\}$$

(22)

$$S_{15,6} = \{1,-\omega^2,\omega,\omega^2,-\omega,\omega^2,$$
$$\omega^2,\omega^2,\omega,\omega^2,-\omega^2,-\omega,\omega^2,\omega,1\}$$

(23)

$$S_{18,6} = \{1,1,\omega^2,-\omega,\omega,\omega,1,-\omega^2,$$
$$-\omega^2,\omega,\omega,-\omega,-1,1,-\omega^2,$$
$$-\omega,\omega,1\}$$

(24)

$$S_{15,4} = \{1,-1,1,i,-i,-1,i,-i,-i,$$
$$1,i,i,1,1,1\}$$

The signals (21) – (23) and (24) are generated by sextic ($m=6$) and quadratic ($m=4$) phase manipulations respectively. Due to this reason ω denotes the 3-th root of the unity, i.e. $\omega^2+\omega+1=0$ and i is the 4-th root of the unity, i.e. $i^2+1=0$.

The coefficients of losses in the SNR of periodic repetitions of the Barker signals (21) – (24) are

$$\gamma_{14,6}=1{,}1693,\ \gamma_{15,6}=1{,}1468,$$
$$\gamma_{18,6}=1{,}1828,\ \gamma_{15,4}=1{,}0775. \qquad (25)$$

These results confirm also the correctness of the new necessary condition (15) for synthesis of efficient SLSFs.

4. Conclusion

The analysis of the factors, which determine the losses in the SNR after the processing of PM signals with two levels PACF by SLSFs, presented in the paper, can be successfully used in the process of development of perspective wireless communication system, employing complex signals in order to enhance radically the effectiveness of the exploitation of the limited natural resource – the electromagnetic spectra.

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