



*Original Contribution*

## МЕТОД ЗА ИДЕНТИФИКАЦИЯ НА ГРЕШКИТЕ И ДИАГНОСТИКА ПРИЛОЖИМ В СИСТЕМИ ЗА АДАПТИВЕН КОНТРОЛ ПОСРЕДСТВОМ ИЗПОЛЗВАНЕ ФУНКЦИИТЕ НА УОЛШ

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## A METHOD OF FAILURE IDENTIFICATION AND DIAGNOSTICS FOR USE WITH THE SYSTEMS FOR ADAPTIVE CONTROL THROUGH THE APPLICATION OF THE FUNCTIONS OF WALSH

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**Abstract:** *The ever increasing interest in the adaptive control, especially in the various aspects of identification and estimation of the subjects of the control is a natural reaction to two existing tendencies in the evolution of the systems for adaptive control. The first of these tendencies is connected with the increasing requirements concerning the higher quality of the control systems, which reflects on the requirements placed on the modeling of the plants of the control used in the synthesis of the systems in question. The second tendency is connected with the considerable progress made in the computer technique and the actual possibilities of its use in the solution of complicated problems in the failure identification and diagnostics. The approach suggested, based on the use of the functions of Walsh is the most effective means for analysis because the practicable application of the Wiener's theory to the non-linear systems considered.*

**Keywords:** *adaptive control, functions of Walsh.*

The ever increasing interest in the adaptive control, especially in the various aspects of identification and estimation of the subjects of the control is a natural reaction to two existing tendencies in the evolution of the systems for adaptive control. The first of these tendencies is connected with the increasing requirements concerning the higher quality of the control systems, which reflects on the requirements placed on the modeling of the plants of the control used in the synthesis of the

systems in question. The second tendency is connected with the considerable progress made in the computer technique and the actual possibilities of its use in the solution of complicated problems in the failure identification and diagnostics.

The knowledge of the current information about the dynamic state of the system functioning enables, on one side, the programming of an optimum control through the adaptation to changing external condition, and on the

other side, taking relevant and timely decisions by failure in the sub-systems of the systems for adaptive control. The adaptation is a mark of complexity of the systems for control that usually perform under conditions of a considerable "a priori" uncertainty. Most often this uncertainty is reduced to uncertainty of the model of the controlled plant. In this case the architecture of such a system includes a sub-system for real-time evaluation of the parameters of the plant model, thus enabling completely adjustment of the parameters of the system by a parametric discord or even changes in the systems structure of control by failures.

The diagnostics of the failures carried out by the sub-system for evaluation makes possibly the switching on of supplementary stages or sub-systems into the circuit of the controlling part of the system of automatic control or even introducing architectural rearrangements with the use of additional sub-systems. The sub-system for evaluation of the performance conditions should possess high speed and relevant sensitivity to be able to generate any given an accurate model of the controlled non-stationary plant.

The plant's identification in a closed circuit of adaptive control imposes certain restrictions by the traditionally employed algorithms for evaluation. This is due to the linearity of the relation between the controlled inputs and the noise-corrected outputs of the plant through the application of a feedback. It is seemed necessary to develop special methods for controlled plants' identification for the purpose of adaptive control. Evidently those

methods should possess the following features: high speed of action, noise proof operation, high sensitivity, possibilities for failure warning and simplicity of algorithms. Those features are a prerequisite for a truthful and reliable diagnostic of the plant performance.

To a great extent, the active statistical method of identification of the plant's dynamic behavior with testing random signal possesses the mentioned merits. They make possibly, if optimal parameters and intensity of signals selected, to probe the plant without disturbing the regime of its performance.

The Wiener theory of non-linear system analysis gives a presentation for the output of the unknown system as a set of orthogonal functions  $C_n[K_n, x(t)]$  where  $\{K_n\}$  is a sequence of nuclei characteristic for the non-linear system;  $x(t)$  is the input of the system. The functional appear orthogonal in the sense, that if at the input of the system is applied a white noise, the following relationship will be valid:  $\theta \{G_n[K_n, x(t)]G_m[K_m, x(t)]\} = 0$ ,  $n \neq m$ , where the symbol  $\theta$  can note averaging for the population.

The sequence of nuclei can be determined through the devising of parallel filters having relevant pulse characteristics and constituting a complete set of orthogonal functions  $[F_n(t)]$ . At the input of the system analyzed and the filter appears white noise  $x(t)$ . The output of the system is multiplied by the outputs of the parallel filters and the resulting signal is applied to the input of the stage which is integrating along the time axis (averaging) – Fig.1

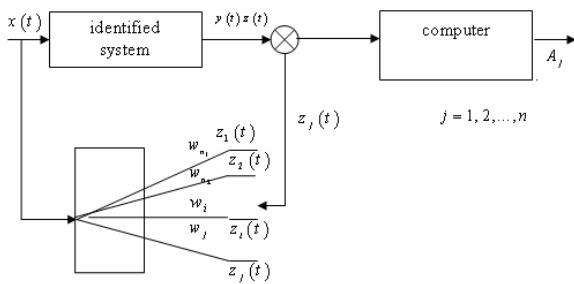


Fig.1

On the assumption, that the system under analysis is a linear one (the method can be extended for non-linear system too) we can obtain for the output signal of the system:

$$(1) \quad y(t) = \int_0^{\infty} x(t-\tau)h(\tau) d\tau,$$

where  $h(\tau)$  is the pulse transfer function of the system plant to determination.

Similarly, the output signal of any  $j$ -th filter will be:

$$(2) \quad z_j(t) = \int_0^{\infty} x(t-\tau)w_j(\tau) d\tau$$

Time-averaging the above indicated signals, the average value  $A_j$  could be obtained, depending on the  $j$ -th and equal to filter

$$(3) \quad A_j = \overline{y(t)z_j(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y(t)z_j(t) dt$$

Based on equations (1), (2), (3) and averaging with the multitude (taking into account the ergodic nature) of white noise  $x(t)$  is obtained:

$$(4) \quad A_j = \int_0^{\infty} \int_0^{\infty} \overline{[x(t-\tau)x(t)]} w(\tau)h(\tau) d\tau d\tau$$

or

$$(5) \quad A_i = \int_0^{\infty} w_j(\tau)h(\tau) d\tau$$

Equation (5) is an integral equation of the identification in terms of  $h(\tau)$ .

Taking into account its orthogonal character, its solution in simplest form is obtained as a converging series:

$$(6) \quad h(\tau) = \sum_{j=0}^{\infty} A_j w_j(\tau)$$

Let the number of the parallel filters is  $n$ . Then as a result of the averaging procedures the coefficient  $C_{P_1 P_2 \dots P_n}$  is obtained. This a coefficient of the dissolution of the nuclei in the form of a sum of  $n$ -fold products of the functions  $F_j(U_j)$ :

$$K_n(U_1, U_2, \dots, U_n) = \sum_{p_1=0}^{\infty} \sum_{p_2=0}^{\infty} \dots \sum_{p_n=0}^{\infty} C_{P_1 P_2 \dots P_n} F_{P_1}(U_1) \dots F_{P_n}(U_n).$$

Let us make use of the properties of the functions of Walsh:

$$(7) \quad \int_0^1 w_n(t_1 \oplus t_2) w(t_1) dt_1 = \int_0^1 w_n(t_1) w(t_1 \oplus t_2) dt$$

which is analogous to the properties of the integral of pressing (contraction).

This analogy, as well as the analogy between the expression above and the Hinchin-Wiener theorem is opted as a basis for the approach outlined for evaluation of the characteristics of non-linear system. The filters of Walsh are equal of the parallel filters from the theory of Wiener, transforming the input  $f(t)$  into an output function:

$$(8) \quad y(t) = \int_0^1 w_u(u) w(t \oplus u) du$$

or

$$(9) \quad y(t) = a_n w_n(t)$$

If  $A$  is the dispersion of the white noise at the input of the filter, the coefficients of the output of the filter have a Gaussian distribution and parameters obtained from the relationship:  $\overline{\{a_n\}} = 0; \overline{\{a_n a_m\}} = A \delta_{n,m}$  where  $\delta_{n,m}$  is the symbol of Kroniker. As  $x(t_1) \dots x(t_n) = 0$  for uneven values of  $n$  and  $\overline{\{x(t_1) \dots x(t_n)\}} = \sum_i \prod_j \overline{\{x(t_i) x(t_j)\}} \delta_{i,j}$  for

even values of  $n$ , where the summing is carried out in terms of all products of pairs of multiplicands, it follows

$\overline{\{a_{p_1}, a_{p_2}, \dots, a_{p_n}\}} = 0$  if  $n$  is uneven, and

$$(10) \quad \overline{\{a_{p_1}, a_{p_2}, \dots, a_{p_n}\}} = \sum_i \prod_j \overline{\{a_{p_i} a_{p_j}\}} \delta_{ij}$$

if  $n$  is even.

Let the signal at the output of the non-linear system is related to the input signal according to the equation:

$$(11) \quad y(t) = \sum_{n=0}^{\infty} G_n [H_n, x(t)]$$

where  $\{G_n\}$  is a set of orthogonal functions by the relevant change of the contraction integral by the operator  $\oplus$  (order of magnitude summation following mod 2). By  $n = 1, 2, 3, \dots$  the following relationships are valid:

$$(12) \quad G_0 [H_0, x(t)] = H_0;$$

$$G_1 [H_1, x(t)] = \int_0^1 H_1(u) du;$$

$$G_2 [H_2, x(t)] =$$

$$\int_0^1 \int_0^1 H_2(u, v) x(t \oplus u) x(t \oplus v) dudv - A \int_0^1 H_2(u, v) du;$$

$$G_3 [H_3, x(t)] =$$

$$\int_0^1 \int_0^1 \int_0^1 H_3(u, v, w) x(t \oplus u) x(t \oplus v) x(t \oplus w) dudvdw -$$

$$-3A \int_0^1 \int_0^1 H_3(u, v, w) x(t \oplus u) dudv.$$

The nuclei  $\{H_n\}$  are obtained through the solution of the result in a series of Walsh-Fourier, following:

$$(13) \quad H_1(u) = \sum_{k=0}^{\infty} C_k w_k(u)$$

$$(14) \quad H_2(u, v) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} d_{ij} w_i(u) w_j(v)$$

If white noise applied to the input of the system, then:  $x(t) = \sum_{k=0}^{\infty} a_k w_k(t)$  and the output signal can be present as (accounting for the orthogonal equations (11), (12), (13) and (14)):

$$(15) \quad y(t) = H_0 + \sum_{k=0}^{\infty} a_k C_k w_k(t) + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_i a_j d_{ij} w_i(t) - A \sum_{j=0}^{\infty} d_{ij} + \dots$$

Making use of the basic multiplication character of the functions of Walsh, the output signal of the system is presented in the form:

$$(16) \quad y(t) = \sum_{k=0}^{\infty} b_k w_k(t)$$

The coefficients  $b_k$  are calculated through comparison with the same coefficients by  $w_k(t), k \neq 0$  using equations (15) and (16).

$$(17) \quad b_k = a_k C_k + \sum \sum a_i a_j d_{ij} + \dots$$

where the asterisk \* denotes a double summation for those indices for which  $i \oplus j = k$ . Assuming that  $k \neq 0$  it follows from the properties of the operator used,

that  $i \neq j$ . For  $k = 0$ :

$$(18) \quad b_0 = H_0 + a_0 c_0 + \sum_{i=0}^{\infty} a_i^2 d_{ij} - A \sum_{i=0} d_{ij} + \dots$$

From the last equation it follows, that  $\overline{\{b_0\}} = H$  because the second term at right hand side part of the equation (18) is zero (according to the equation (10)),  $\overline{\{a_0\}} = 0$  and  $\overline{\{a_i\}} = A$ . The remaining terms of the equation are also equal of zero because of the same reasons.

As evident from Fig.2, the coefficients of Walsh-Fourier are obtained through the equation:  $C_n = \overline{\{a_n b_n\}} / A$

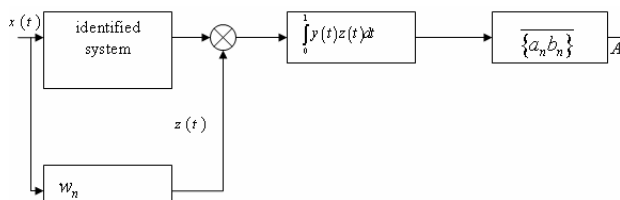


Fig.2

As a proof of this relationship, let us consider the output of filter of  $z(t)$ .

Following equation (3) we can write  $z(t) = a_n w_n(t)$ : thus

$$\int_0^1 y(t) z(t) dt = \sum_{k=0}^{\infty} b_k a_n \int_0^1 w_k(t) w_n(t) dt.$$

As  $\{w_n(t)\}$  is an orthogonal set of functions in the interval  $[0,1]$  the above relationship is reduced to

$$\int_0^1 y(t) z(t) dt = a_n b_n. \text{ From equation (17)}$$

with  $k=0$  is obtained:

$$\overline{\{a_n b_n\}} = \overline{\{a_n b_n\}} C_n + \sum \sum \{a_i a_j a_n\} d_{ij} + \dots$$

therefore, accounting for equation (10) is obtained:  $\overline{\{a_n b_n\}} = A C_n$ , which is the proof needed. In Fig.3 is presented the illustration of method for determination of nuclei of second order  $H_{2(u,v)}$ .

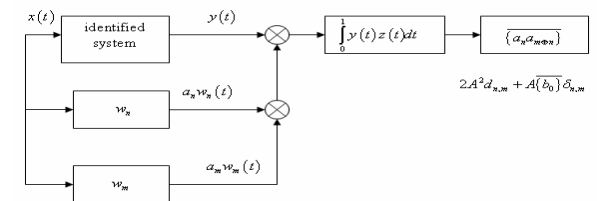


Fig.3

The coefficients  $H_2(u,v)$  are found from the equation:

$$d_{n,m} = \frac{1}{2} \left[ \frac{\overline{\{a_n a_m b_{n \oplus m}\}}}{A^2} - \frac{\overline{\{b_0\}}}{A} \delta_{nm} \right].$$

The proof for the result indicated above is based on the use of the multiplicative and orthogonal characteristics of the function of Walsh. The approach suggested, based on the use of the functions of Walsh is the most effective means for analysis because the practicable application of the Wiener's theory to the non-linear systems considered. The current information received in the process of identification can be used not only in the self-adjustment of the system, but for diagnostics of the failures and prevention of critic-situations. Both problems, the self-adjustment and control of the state of the system carried out by the sub-system for estimation of the subject and can be solved within the possibilities offered by a specialized numerical computing device (NCD). Such an adaptive system for automatic control and identification of the dynamic characteristics of the



plant controlled should make use of combined principle of control: adaptive control, by a relatively slow drift of parameters of the changes in of the configuration of the controlling part or by a step-like change of the parameters following a failure, in some subsystem. The state of system for control is described by a "so called" vector of identification parameters of the closed circuit system. An example is the coefficients of the Walsh-Fourier series in the sequence of nuclei obtained in the outlined approach. Besides, it is necessary to select the components of the vector of identification according to the criteria for maximum sensitivity of the system towards changes of its parameters. The problem concerning the classification of the states of the system can be solved based on the application of the theory of image identification: it is necessary to consider the set of identifiable parameters  $A$  based on one defined (in advance) prognosis (slate). The set of slates of the system  $R$  is broken into sub-sets  $Q_i; Q_0$  – a set of states, corresponding to a working system,  $Q_i; i = 1, 2, \dots, N$  set of states corresponding to a failed (non-working) system, due to a failure of the  $i$ -th sub-system. The diagnostic is carried out, based on the distance of the current vector  $A$  to the vectors corresponding to the particular sub-sets  $A_{Q_0}, A_{Q_i}$  or based on the distance to the standards  $A^*_{Q_0}, A^*_{Q_i}$ , the co-ordinates of which are the average of the coordinates belonging to a defined set with an approach like the outlined above, all solutions concerning any possible failure, should be fore-

seeing advance. Besides, it is necessary to select the most informative identifiable parameters, their order, as well as the question concerning the coding of the vector of the identifiable parameters, aiming at defining the solution – logical function used in the classification of the states of system. The use of the algorithm with the computing device, presupposes the fulfillment of the following sequence of operations:

- Periodical measurement of the identifiable parameter;

- Classification of the states of the system (finding of the solving logical function);

- Generating of the signals for self-adjustment of the system (availability of a set of parameter for self-adjustment I for control of plant and controlling system) by an absence of failures.

Generation of controlling actions makes the relevant switching active in the controlling part of the system by a failure. The approach to the adaptive systems for control is very effective by the design of numerical (multi-processor) system for control, plant to rigid requirements for high reliability changes into the system's structure, effecting the connections between the input devices, the processors, the elements of the memory, the output devices. The controlling part and the subsystem for evaluation can be designed as a multi-processor controlling complex.

### References

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