Journal scientific and applied research, vol. 15, 2019
International Journal

Original Contribution ISSN 1314-6289

CRITERION FOR DEFINING THE PROBABILITY ERROR WHEN DISCOVERING DISTANT OBJECTS BY MEANS OF OPTIC-ELECTRONIC DEVICES

Stiliyan Stoyanov

SPACE RESEARCH AND TECNNOLOGY INSTITUTE
BULGARIAN ACADEMY OF SCIENCES, SOFIA 1113,
ACAD. GEORGI BONCHEV ST., BL.1
E-mail: stil717@yahoo.com

ABSTRACT: *The aim to practically use the results from the observations for discovery of distant objects in the visible and the infrared part of the optical specter means to seek a definite answer about a signal presence which would answer the question about the presence in the controlled space of the sought object. On research basis the signal presence probability can be defined which is described by means of a likelihood ratio. The initial value of the likelihood ratio does not contain any unknown values. It is defined on an experimental basis by using devices with concrete tactical and technical characteristics in a concrete environment by considering the fluctuations probability.*

KEY WORDS: *discovery criterion, probability mistake, distant objects*

In order to find distant objects and to exactly define their characteristics, time is a factor with great significance. Such kind of research is current for the ecological sphere [2, 3, 4, 5, 7] and also for the demands of the defense [1, 5, 6].

The tasks for a practical implementation of the observation results are looking for a certain result – presence of a signal, which allows the solution of the problem with the presence of the controlled space of the searched object, aiding (causing) the signal appearance [8]. On the basis of observations, the probability of signal presence can be found and it is expressed by the likelihood ratio.

If the parameters of the signal are known beforehand and the a priori probability for signal presence is also known, we can consider as almost obvious the criterion for the likelihood ratio, according to which the result for signal presence is valid when the likelihood ratio $A_{aop}\, A_{op} \geq A_{aop}$, where A_{aop} is a threshold of the absolute likelihood ratio.

The threshold of the likelihood ratio is defined on the basis of experiment by using observation devices in a certain environment which reports probabilities and fluctuations. There are two versions when discovering distant objects:

- We decide that there is a signal and actually the signal is correctly discovered or the signal is wrong;
- We decide that there is no signal and actually the signal is missed or we are right in our findings that there is no signal.

There is one wrong answer in the two versions. The error is when there is no signal and we consider that there is one. This error is defined as false signal. The second error occurs if there is a signal and we consider that there is not. This error is considered as an omission and it occurs when the signal from the discovered distant object is weak. When we calculate the distribution of the density probability when there is actual signal presence and when there is actual signal absence, we can calculate the probability of the error occurrences. The conditional density probability of the value A when there is signal presence is marked with $\omega_A(A_{op}/S)$, and at signal absence – with $\omega_A(A_{op}/0)$.

The conditional probability for false signal, i.e. the probability of the situation "a signal is present" when in fact there is no signal, is defined by the equation:

(1) $$P_{PS} = \int_{A_n}^{\sim} \omega_A(A_{op}/0)dA_p .$$

The conditional probability for omission is:

(2) $$P_{AP} = \int_{-\sim}^{A_n} \omega_A(A_{op}/S)dA_p .$$

We should mention that the received probabilities are conditional and not absolute because the conditional probability for the omission gives the average ratio of the number of omitted objects to the total number of objects which are in the visual field. The unconditional probabilities for false signal P_{bps} and omission P_{bnp} will be conditionally equal to the multiplication of the a priori probabilities for presence and absence of signal:

(3) $$P_{bps} = P_{PS}P(0),$$

(4) $$P_{bnp} = P_{np}P(S).$$

Usually in the problems for discovery of distant objects, the value *P(S)* is unknown but practically the conditional probability of omission, equal to the relative part of the omitted signals is much more important than the conditional one and this is the reason why it is advisable to be used for characterizing the activities of the systems for distant objects discovery.

If the a priori probability *P(S)* $\approx$ 1, it means that the unconditional probability for false signal $P_{bps} = P(0)P_{PS} \approx P_{PS}$, i.e., it will practically coincide with the conditional probability. The purpose of using the unconditional probability for registering a false signal is that it gives the number of false noises with the exception of the cases when the noises coincide with the signal.

According to the criterion for the perfect observer, the observation system should be one that it gives the minimum full (total) probability for an error:

(5) $$P_{bnp} + P_{bps} = min.$$

We will go through the process of finding the correct resolution with the help of this criterion. For this purpose, first we will calculate the probability of error when the density of the noise distribution is known. Taking into account (3), the unconditional probability for a false signal is received:

(6) $$P_{bps} = P(0)\iint\limits_{G_\alpha} \omega_N(Y)dy_1...dy_H.$$

The integration is done according to a still unknown area G_α. When we define it, we will understand at what values on the ordinate of realization we will get solutions for presence and at what values – absence of signal.

The probability of omission is equal to:

(7) $$P_{bnp} = P(S)\iint\limits_{G_\beta} \omega_N(Y-S)dy_1...dy_H .$$

In this case the area of integration G_β includes the area from every possible value of *Y*, which lies outside the area G_α.

If we find that the integral from the density of the probability throughout the whole area from possible values of the variables at integration equals one, instead of (7) we can write:

(8) $$P_{bnp} = P(S)\left[1 - \iint\limits_{G\alpha} \omega_N(Y-S)dy_1...dy_N\right].$$

Adding the probable errors (6) and (8) and taking into account that the integration area is equal, we get:

(9) $$P_{bps} + P_{bnp} = P(S) + \iint\limits_{G\alpha} [P(0)\omega_N(Y) - P(S)\omega_N(Y-S)]dy_1...dy_H = min.$$

We observe a minimum of probable errors in the area $G\alpha$, defined by the values of Y_n, from which:

(10)
$$P(0)\omega_N(Y_n) - P(S)\omega_N(Y_n - S) = 0.$$

or

(11)
$$\frac{\omega_N(Y_n - S)}{\omega_N(Y_n)} = \frac{P(0)}{P(S)},$$

i.e.

(12)
$$A_{op} = \frac{P(0)}{P(S)}$$

or

(13)
$$A_{aop} = \frac{P(S)\omega_N(Y_n - S)}{P(0)\omega_N(Y_n)} = 1.$$

The solution of presence – there is an object, refers to a situation when *Y* is situated in the area G_α.

This is the way how the criterion for the perfect observer refers to the criterion for likelihood ratio but since the value of this threshold is usually undefined, actually this criterion dos not offer any additional information.

There is also a criterion for minimum weighed probability error, according to which the probability P_{PS} and P_{np} are taken in a certain weight ratio:

(14)
$$P_{np} + cP_{PS} = min.$$

Analogous to the preceding example, it can be shown that this criterion is also equated with the criterion of the likelihood ratio and the value of the coefficient *c* here is also not defined.

We will also look at the criterion of Neyman–Pearson according to which the acceptable values of the conditional probability for a false signal in the system should be equal to the given value *k*, and the conditional probability for omission will be minimum:

(15)
$$P_{PS} = k, \qquad P_{np} = min.$$

In order to accept the correct solution on the basis of this criterion, it is necessary to find the minimum of P_{np} in the area of integration for which we have the equation:

(16) $$\iint\limits_{G\alpha} \omega_N(Y)dy_1...dy_H = k \,.$$

In this case, applying the method of the Lagrange undetermined multipliers, we have to find the minima of the function:

(17) $$P_{np} + \lambda P_{PS} \,,$$

and to define λ according to the condition (15). When calculating (17) we get:

(18) $$P_{np} + \lambda P_{PS} = \left[1 - \iint\limits_{G\alpha} \omega_N(Y-S)dy_1...dy_H\right] + \lambda \iint\limits_{G\alpha} \omega_N(Y)dy_1...dy_H =$$
$$= 1 + \iint\limits_{G\alpha} [\lambda\omega_N(Y) - \omega_N(Y-S)]dy_1...dy_H = min,$$

from which it follows that the minima will be received when integrating at the area of the negative values of the equations:

(19) $$\lambda\omega_N(Y) - \omega_N(Y-S) \,,$$

i.e. at the area where:

(20) $$A_{op} = \frac{\omega_N(Y-S)}{\omega_N(Y)} \geq \lambda \,.$$

The value of the undetermined multiplier λ can be determined by the condition:

(21) $$\iint\limits_{\frac{\omega_N(Y_n-S)}{\omega_N(Y_n)}=\lambda} \omega_N(Y)dy_1...dy_H = k \,,$$

from where we get:

(22) $$A_{op} = \lambda(k) \,.$$

Consequently, in this situation it is also necessary to calculate the likelihood ratio A_{op} and if it is more than a certain charge value, then an answer can be given for a signal presence and if it is less – then there is no signal.

It can be concluded that the Neyman–Pearson criterion is the most convenient for practical usage from all the criteria mentioned above. Especially if we use acceptable frequency of repetition of the false signal in time instead of the acceptable probability for a false signal.

References:

[1]. Andonov A. Synthesis of signals with discrete modulation, not causing transitional processes in linear systems, Research papers journal of Anniversary scientific session “100 years since the brothers Wright flight”, National Military University V. Leski, Aviation Faculty, Dolna Mitropolia, part I, 2003.

[2]. Getsov P. Space, Ecology, Security, New Bulgarian University, Sofia, 2002, 211 p.

[3]. Getsov P. Satellite systems for ecological monitoring, International conference Energetics and environment protection: regional problems, Sofia, 2000, p. 5–9.

[4]. Getsov P., G. Mardirossian, Zh. Zhekov, Space Research Institute – BAS – Facing the practical ecological problems, Scientific papers journal part III, Scientific conference 2000, HMSAACD P. Volov, Shumen, 2000, p. 49–53

[5]. Getsov P., P. Penev, The aerospace technologies and the military factor for the national security, Scientific conference, HMSAACD P. Volov, Shumen, 2000, p. 20–30.

[6]. Getsov P. Seminatural design of systems for flight control of unmanned aircraft, Acad. Publishing house Marin Drinov, Sofia, 2003, 201 p.

[7]. Mardirossian, G. Aerospace methods in ecology and study of environment, Acad. Publishing house Marin Drinov, Sofia, 2003, 201 p.

[8]. Stoyanov S. Mathematical model for an optical tract of optical-electronic device, Research papers journal from Anniversary scientific session “100 years since the brothers Wright flight”, National Military University V. Leski, Aviation Faculty, Dolna Mitropolia, part II, p. 151–156 .