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DETERMINATION OF THE ANGULAR COORDINATES OF ASTRONOMICAL OBJECTS

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ABSTRACT: *Among the methods for angular coordinates determination of the astronomical objects, particularly a topic of the present day is the Turner method based on static processing of data information for three and more stars number.*

With regard to used in astronomical practice optical electronic apparatuses which have high resolution and comparatively closely a field of vision lower than one degree of Celsius, it grows up the actuality of the methods for angular coordinates determination at minimum number of supporting (catalogue) stars. In this conditions it's necessary to be re-examine the respect to orthogonality methods, the accuracy of which can be increase through statistical analyze.

Calculation for ideal coordinates and the equatorial coordinates of an astronomical object are shown as well as the possible errors for their determination.

A priori it is supposed the orthogonality method is more effective because of the fact that the possibility for appearance in the two stars field of vision is bigger than of the three ones.

KEY WORDS: *angular coordinates*

Among the methods for angular coordinates determination of the astronomical objects, a topic of the present day particularly is the Turner method based on statical processing of data information for three and more supporting stars number $k \geq 3$ and the method of four constant (orthogonal) ones using two supporting points [1].

In connection with the widely spread optic-electronical measuring systems in the astronomical practice, which have high-resolution ability and comparatively a narrow field of vision [2, 3, 4], the actuality of the methods for determining the angular coordinates arises in minimum of number points of support (catalog) stars. In these conditions the attitude to the orthogonal methods has to be reconsidered, the accuracy of which can be arised by statistical arrangement way in $k > 2$.

The influence of the differential effects for the optical systems with a narrow field of vision is insignificant [5, 6, 7].

A method for angular coordinates determination of the astronomical objects, spreading on stars number $k > 2$ and objects coordinates calculation in methods of least squares is presented.

The correlation between the ideal and the measured coordinates is being putting into formulas [1,3]:

(1) $$\begin{aligned} \zeta_1 &= ax_i + by_i + c \\ \eta_i &= -bx_i + ay_i + f \end{aligned} \qquad \text{if } i = 1, k$$

where: x, y – calculated coordinates of supporting stars;

ξ, η – ideal coordinates of supporting stars.

The expression (1) can be done also in the following way:

(2) $$\nu_{1.i} = x_{1,i}a_1 + x_{2,i}a_2 + 1a_3 + 0a + \xi_{1,i}$$

or in a matrix form:

(3) $$\vec{v} = \overrightarrow{Xa} + \vec{\xi}$$

when: X is a matrix made by rows

(4) $$\vec{x}_{1,i}\{\vec{x}_{1,i}, \vec{x}_{2,i}, 1, 0\} \quad \text{and} \quad \vec{x}_{2,i}\{\vec{x}_{2,i} - \vec{x}_{1,i}, 1, 0\}$$

and calculating the equations

(5) $$Q = X'X, K = Q^{-1} \quad , \quad \vec{L} = X'\vec{\xi}, \vec{a} = -K\vec{L} \quad ,$$

when: $\vec{a}$ – vector evaluating the constant staff;

Q – a matrix with normal equations as follows:

(6) $$\begin{matrix} [x_1^2]+[x_2^2] & 0 & [x_1] & [x_2] \\ 0 & [x_1^2]+[x_2^2] & [x_2] & -[x_1] \\ [x_1] & [x_2] & k & 0 \\ [x_2] & -[x_1] & 0 & k \end{matrix}$$

The cell method for the matrix calculation [8] with regards to the symmetry and other characters of the matrix Q applying, it's allowed to find the mostly suitable form for the passage from the calculated coordinates to the matrix K elements forming and transporting the matrix X and forming and rotating the matrix Q. The matrix K is as follows:

(7) $$K = Q^{-1} = (X'X)^{-1} = \frac{1}{W}$$

when: $$R = \left[x_1^2\right] + \left[x_2^2\right] ;$$

$$W = kR - \left[x_1^2\right] - \left[x_2^2\right] .$$

For the vector $\vec{L}$ elements are made the values as follows:

(8) $$\vec{L} = X'\vec{\xi} = \begin{vmatrix} [x_{1,}\xi_1] + [x_2\xi_2] \\ [x_2\xi_1] + [x_1\xi_2] \\ [\xi_1] \\ [\xi_2] \end{vmatrix}$$

For ideal coordinates of the astronomical object can be get the expression:

$$\vec{a} = -KL \ ,$$

(9) $$\xi_1 = \vec{x}_1, \vec{a} \ ,$$

$$\xi_2 = \vec{x}_2, \vec{a} \ ,$$

and its equatorial coordinates estimates at known formula [1]:

(10) $$\alpha = arctg\left(\frac{\xi_1}{\cos D - \xi_2 \sin D}\right) + A$$

(11) $$\delta = arctg\left[\frac{(\sin D + \xi_2 \cos D)\cos(\alpha - A)}{\cos D - \xi_2 \sin D}\right] ,$$

when A and D are the equatorial coordinates in the stuff centre.

The errors in receiving coordinates of the object Δ_α and Δ_δ are collected from the error in the reduction Δ_ξ and the error of the calculated coordinates of the object Δ_x.

(12) $\dot{\Delta}_\alpha = \dot{\Delta}_{\xi_1} + M\dot{\Delta}_{x_1}$ and $\dot{\Delta}_\delta = \dot{\Delta}_{\xi_2} + M\dot{\Delta}_{x2}$

Here the scale multiplier $M = \sqrt{a_1^2 + a_2^2}$ [1].

The reduction error in the solving of a system of 2k equations of four unknowns in the least squares method is determined by dispersion [9]:

(13) $$\sigma^2\left(\dot{\Delta}_{\xi_1}\right) = \frac{\lfloor v_1^2 \rfloor}{2(k-2)} P_1 \quad , \quad \sigma^2\left(\dot{\Delta}_{\xi_2}\right) = \frac{\lfloor v_2^2 \rfloor}{2(k-2)} P_2$$

when the weight coefficients P_1 and P_2 are determined by formulas [11]:

(14) $$P_1 = \sum \left(\frac{\partial \xi_1}{\partial a_j}\right)^2 K_j + 2\sum \left(\frac{\partial \xi_1}{\partial a_i}\right)\left(\frac{\partial \xi_1}{\partial a_j}\right) K_i \ ;$$

(15) $$P_2 = \sum \left(\frac{\partial \xi_2}{\partial a_j}\right)^2 K_j + 2\sum \left(\frac{\partial \xi_2}{\partial a_i}\right)\left(\frac{\partial \xi_2}{\partial a_j}\right) K_i \ .$$

Having an attention to $\frac{\partial \xi}{\partial a} = \vec{x}$, P_1 and P_2 can be presented in a square form suitable of the vectors $\vec{x}_1$ and $\vec{x}_2$ and the matrix K [8]:

(16) $$P_1 = \overrightarrow{x_1}' K \overrightarrow{x_1} \quad \text{and} \quad P_2 = x'_2 K x_2$$

The final formula view for the accuracy estimation is:

(17) $$\tilde{\sigma}\left(\dot{\Delta}_\alpha\right) = \sqrt{\frac{\lfloor v_1^2 \rfloor}{1(k-2)} P_1 + M^2 \sigma^2\left(\dot{\Delta}_{x_1}\right)} \cos D$$

(18) $$\tilde{\sigma}\left(\dot{\Delta}\delta\right) = \sqrt{\frac{\lfloor v_2^2 \rfloor}{1(k-2)} P_2 + M^2 \sigma^2\left(\dot{\Delta}_{x_2}\right)}$$

In conclusion it has to be marked the question for comparison of the effectiveness of that and the other methods can be a topic of special investigation but apriori it is supposed the orthogonality method is more effective because of the fact that the possibility for appearance in the two stars field of vision is bigger than of the three ones.

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