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DISCOVERY OF DISTANT OBJECTS BY MEANS OF OPTIC-ELECTRONIC DEVICES, LIKELIHOOD RATIO WHEN THE SIGNAL IS ABSOLUTELY UNKNOWN

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ABSTRACT: *The signal registration from distant objects is one of the major problems which concerns many scientists who work in the field of the space physics and the distant methods in the visible and the close infrared part of the optic spectre.*

For most of the systems which discover distant objects, a very small a priori probability for presence of a signal is characteristic and the a posteriori information becomes proportional to the likelihood ratio.

To solve practical problems, the probability relation is conveniently presented by means of dividing the density to the discovery probability.

KEY WORDS: *distant objects discovery, likelihood ratio.*

The registering of signals from distant objects at different background brightness is one of the major problems which concern a wide spectrum of scientists who work in the sphere of space physics, distant methods, ecology [1, 2, 3, 4, 5, 6, 7], as well as in the sphere of defence [8, 9].

If there is not any a priori information about the presence of a signal, and this is a common situation when searching and finding, then the only thing which can be concluded from the observations is the likelihood ratio. Sometimes in such situations we assume that the presence and the absence of a signal are equally probable, i.e

$$P(S) = P(O),$$

where: $P(S)$ – a posteriori probability for signal presence;

$P(O)$ – a posteriori probability for signal absence.

It turned out that the likelihood ratio absolutely characterises the probability of signal presence in the experiments.

An extremely small a priori probability for signal presence is characteristic of most of the systems for observation, i.e. *P(S)* << 1, and the a posteriori probability becomes proportional to the likelihood ratio.

In order to solve practical problems, it is convenient to express the likelihood ratio by means of distribution of the probability density of the experiments. First the likelihood to obtain realization is considered when there is no signal *P(Y/O)*, i.e. when the experiment *y(t)* is only noise.

During the experiment for a period of time τ, with interludes of *H* intervals, and at the end of every interval (at points $t_1,\ t_2\ \ldots,\ t_i\ \ldots,\ t_H$) reports are recorded $y_i = y(t_i)$, with corresponding for the given situation values of the noise at moments of time t_i. All the records for the time τ form the kind of *Y,* which can be seen as *H*-dimensional vector with components y_i, where *i* = 1, 2, …, *H,* and the noise value in every moment t_i is within the range y_i to $y_i + d_{yi}$, i.e:

(1) $$y_1 < n_i < y_1 + d_{yi}\ .$$

When using the information from the probability theory about the ratio between the probability *P* and the density probability $\omega(x)$ of a random variable *X* with possible values *x*, we get:

(2) $$\omega(x) = \lim_{\Delta x \to 0} \frac{P(x < X < x + \Delta x)}{\Delta x},$$

and at multidimensional distribution:

(3) $$\omega(x_1, x_2, ..., x_n) = \lim_{\Delta x_1, ... \Delta x_n \to 0} \frac{x_1 < X_1 < x_1 + \Delta x_1, ..., x_n < X_n < x_n + \Delta x_n}{\Delta x_1 \Delta x_2 ... \Delta x_n}\ .$$

On the basis of (3) the following is derived:

(4) $$P(Y/O) = \omega_n(y_1, y_2, ..., y_H) dy_1 dy_2 ... dy_H\ .$$

The following short version of the equation can be written:

(5) $$P(Y/O) = \omega_N(Y)dY,$$

where: $dY = dy_1 dy_2 ... dy_H$ – element of the volume from the H-dimensional space;

$\omega_N(Y)$ – H-dimensional distribution of the probability of the noise.

Using analogy, the likelihood of presence of Y in an element from the volume dY when there is a signal is:

(6) $$P(Y/S) = \omega(Y/S)dY,$$

where: $\omega(Y/S)$ – conditional density probability for realization Y for a mixture of the signal with the noise.

Considering formulas (5) and (6), the equation for the relationships of the conditional probabilities for realization, which are defined as likelihood ratios, it is marked by A_p and we get:

(7) $$A_p = \frac{\omega(Y/S)}{\omega_N(Y)}.$$

Often the mixing of the signal with the noise represents by itself an algebraic sum, i.e. signal and additive noise. It is a common scenario in practice:

(8) $$y_i = n_i + S_i,$$

where: n_i – value of the noise at a moment of time t_i;

S_i – value of the signal at the same moment of time.

In this case the probability of realization of the value y_i coincides with the probability of the noise with the values $n_i = y_i - S_i$. That means that the probability to receive an ordinate y_i in realization which contains a signal coincides with the probability to receive an ordinate $y_i - S_i$ in a realization which contains only noise. Therefore, we get the following at additivity of signal and noise:

(9) $$\omega(Y/S) = \omega_N(Y - S),$$

And from here:

(10) $$A_p = \frac{\omega_N(Y-S)}{\omega_N(Y)}.$$

For a likelihood ratio of a signal with unknown parameters we will consider a situation when the signal depends on certain parameters $A_1, A_2,..., A_k$ and the time, i.e. it can be described as:

(11) $$s(t, A_1, A_2,..., A_k),$$

And these parameters are not absolutely unknown; we do not know the shape of the signal and we know only the realization *Y*. We suppose that the perimeters of the signal have a random characteristics a_k and we know the probability density ω_A of their value at a certain moment of time:

(12) $$\omega_A(a_1, a_2,..., a_k).$$

Then the dependency of the likelihood ratio *P(Y/O)* by the signal does not depend on $\omega_N(Y)dY$, and the probability *P(Y/S),* which was achieved at the given realization at the presence of a random possible signal, is defined by every possible value of the parameters.

In order to define *P(Y/S)* we use the formula of the full probability for a certain event *B*, which occurs with probability $P(B/A_j)$ with the condition to realize the event A_j, which occurs only with a certain probability $P(A_j)$:

(13) $$P(B) = \sum_j P(A_j)P(B/A_j),$$

and hence $\sum_j P(A_j) = 1$.

In this case for the event A_j, all the parameters of the signal fall into the given system in the interval

(14) $$a_{j1} \le A_1 < a_{j1} + \Delta a_{j1},..., a_{jk} \le A_K < a_{j\kappa} + \Delta a_{j\kappa},$$

and so the full set of indexes $(j_1, j_2,..., j_k)$ is necessary to ensure the full coverage of the k-dimensional area of the possible values of the parameters

$A_1, A_2, ..., A_k$. Due to this the probability $P(A_j)$ is expressed by the density of the parameters distribution $\omega_A(a_1, a_2, ..., a_k)$:

(15) $$P(A_j) = \omega_{Aj}(a_{j1}, a_{j2}, ..., a_{jk}).\Delta a_j, \Delta a_{j2}...a_{jk}.$$

The conditional probability $P(B/A_j)$ is by itself the likelihood of the realization *Y* to fall into an element of the volume *dY*, on the condition that the parameters lie in a set interval and they have exactly set values $a_1 = a_{j1}, a_2 = a_{j2}, ..., a_k = a_{jk}$:

(16) $$P(B/A_j) = P(Y/a_{j1}, a_{j2}, ..., a_{jk}) = \omega(Y/a_{j1}, a_{j2}, ..., a_{jk})dY.$$

If we substitute (15) and (16) into (13) we can see that the sum in (13) looks like an interval and when the intervals $\Delta a_{j1}, \Delta a_{j2}, ..., \Delta a_{jk}$ are close to zero, it turns into a k-fold interval:

(17) $$P(Y/S) = dY \iint_k \omega(Y/a_1, a_2, ..., a_k)\omega_A(a_1, a_2, ..., a_k)da_1 da_2...da_k,$$

and the likelihood ratio becomes:

(18) $$A_p = \frac{\iint_k \omega(Y/a_1, a_2, ..., a_k)\omega_A(a_1, a_2, ..., a_k)da_1 da_2...da_k}{\omega_N(Y)}.$$

If we express the relationship:

(19) $$\frac{\omega(Y/a_1, a_2, ..., a_k)}{\omega_N(Y)} = A_p^{|}(a_1, a_2, ..., a_k),$$

we get the final equation for the likelihood ratio:

(20) $$A_p = \iint_k A_p^{|}(a_1, a_2, ..., a_k)\omega_A(a_1, a_2, ..., a_k)da_1 da_2...da_k,$$

where: $A_p^{|}(a_1, a_2, ..., a_k)$ characterises the probability of signal presence with certain parameters $(a_1, a_2, ..., a_k)$.

We can conclude that with random parameters $(A_1, A_2, ..., A_k)$, which have density probability $\omega_{Aj}(a_j)$, their integration according to all of their values gives the mathematical expectation or average value of the likelihood ratio. This is the way (20) defines the probability for signal presence at different possible values of its random parameters.

References:

[1]. Boychev B. Small two-channel photomers from the projects "Active" and "APEX", Aerospace Research in Bulgaria Magazine, issue 14, 1998, p. 92–97.

[2]. Getsov P. Space, Ecology, Security, New Bulgarian University, Sofia, 2002, 211 p.

[3]. Getsov P. Seminatural design of systems for flight control of unmanned aircraft Space Research Institute – Bulgarian Academy of Sofia, 2002, p. 380.

[4]. Getsov P. Satellite systems for ecological monitoring, International conference Energetics and environment protection: regional problems, Sofia, 2000, p. 5–9.

[5]. Getsov P., G. Mardirossian, Zh. Zhekov, Space Research Institute – BAS – Facing the practical ecological problems, Scientific papers journal part III, Scientific conference 2000, HMSAACD P. Volov, Shumen, 2000, p. 49–53.

[6]. Getsov P. The Bulgarian space research and the white book of the European union, Scientific papers journal Space, Ecology, Security, Varna, 2005, p. 7–12.

[7]. Mardirossian, G. Aerospace methods in ecology and study of environment, Acad. Publishing house Marin Drinov, Sofia, 2003, 201 p.

[8]. Getsov P. Getsov P. Seminatural design of systems for flight control of unmanned aircraft Space Research Institute – Bulgarian Academy of Sofia, 2002, p. 380. Acad. Publishing house Marin Drinov, Sofia, 2011, 201 p.

[9]. Getsov P., P. Penev, The aerospace technologies and the military factor for the national security, Scientific conference, HMSAACD P. Volov, Shumen, 2000, p. 20–30.

[10]. Enoch J. M. Effect of the Suze of a complex display upon visual search, JOSA, 1989, 49, p. 280 – 286.

[11]. Tzekova V., Tzekov E. Bright spots selection in TV – images, Aerospace Research in Bulgaria, vol 27, 2003, p.p. 151-156.