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ABOUT THE CHOICE OF PARAMETERS WHEN REPRESENTING THE NORMAL EARTH AS A REFERENCE FRAME IN GEODESY

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Abstract: *When using the Normal Earth as a reference frame, it is allowed to choose its parameters approximately, provided that there will be no difficulties in linearizing the boundary values. In many cases, however, it is convenient to find specific solutions corresponding to the selection of the Normal Earth parameters best suited to the real Earth. Thus, it is of interest to determine, as far as possible, the exact values of these parameters.*

Keywords: *normal field, normal gravity, frame of reference.*

Introduction

The main differences between the real figure of the Earth and its gravitational field and the Normal Earth are characterized by the coefficients of the expansion of the gravitational potential of the Earth in an order of spherical functions and the height of the quasi-geoid above the general Earth ellipsoid. The concept of a Normal Earth and the related concept of a normal gravitational field are largely arbitrary, depending on how they are used to solve problems in geodesy, geophysics, astronomy, and other scientific fields.

n geodesy, the Normal Earth is presented in the form of a body whose outer surface is a general terrestrial ellipsoid - a rotational ellipsoid, which is an equipotential surface for the normal potential. The value of the force of gravity determined by the potential on this equipotential surface is called the normal value of the force of gravity [2]. When measuring the gravitational field of the Earth, it is always determined by reference, i.e. relative to a given value of gravity which is considered normal.

When using the Normal Earth as a reference frame, it is allowed to choose its parameters approximately, provided that there will be no difficulties in linearizing the boundary values. In many cases, however, it is convenient to find specific solutions corresponding to the choice of Normal Earth parameters best suited to the real Earth [1].

1. Choice of Normal Earth parameters

If geodynamic phenomena such as the uneven rotation of the Earth around its axis, the change in the position of the axis of rotation and the center of mass are neglected, the first two conditions can be formulated:

1) the center of the rotation ellipsoid must coincide with the center of gravity of the Earth, and the main inertial axis (axis of rotation) – with the axis of rotation of the Earth;

2) the angular rates ω of rotation of the geocentric rotation ellipsoid and the real Earth must be the same;

The subsequent conditions for the selection of the parameters of the Normal Earth are closely related to the solution of the problems of celestial mechanics, in which the most common representation of the potential of attraction of the masses of the Earth (including its atmosphere) is in the form of decomposed into a series of spherical functions geocentric coordinates r, Φ and L.

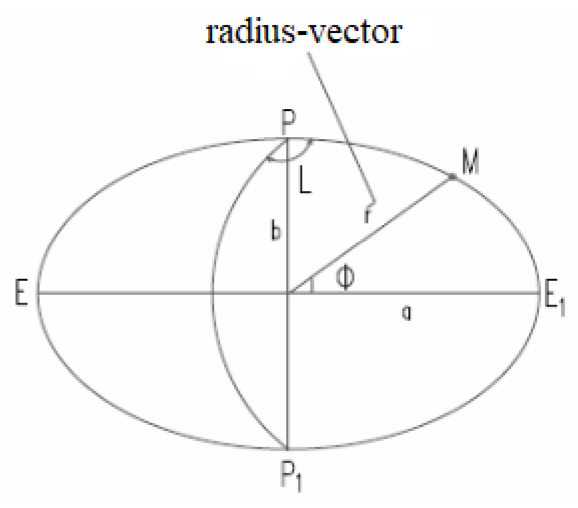

Fig. 1. Geocentric coordinate system [1]

$$V=\frac{fM}{r}\left[1+\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n\left(C_{nm}\cos mL+S_{nm}\sin mL\right)P_{nm}\left(\sin\Phi\right)\right], \quad (1)$$

where fM is the geocentric gravitational constant, the product of the universal gravitational constant f and the mass of the Earth M;

S_{nm}, C_{nm} - spherical geopotential coefficients defining the distribution of masses in the Earth's crust;

a_e - a linear parameter that is introduced so that the coefficients C_{nm} and S_{nm} are dimensionless quantities.

The quantity a_e is taken to be equal to the semi-major axis or the equatorial radius of the rotational ellipsoid - the surface of the Normal Earth.

$P_{nm}(\sin\Phi)$ defines the Legendre polynomial of n-th degree and m–th order. The products $P_{nm}(\sin\Phi)\cos mL$ and $P_{nm}(\sin\Phi)\sin mL$ represent the corresponding spherical functions, and the quotient r^{n+1} represents spherical functions of geocentric coordinates.

The gravity potential *W* is equal to the sum of the drag potential *V* and the centrifugal force potential *Q*, which in the accepted notation is equal to:

$$Q=\frac{\omega^2 r^2}{2}\cos^2\Phi \quad (2)$$

Due to the fact that the origin of the coordinate system coincides with the center of gravity of the Earth in equation (2) there are no harmonic coefficients of the first degree. Spherical functions of zero order $P_{n,0}(\sin\Phi)$ do not depend on the geographic longitude and are called *zonal*. The zonal harmonic coefficient $C_{2,0}$ is the largest absolute harmonic coefficient of the geopotential. According to modern data, it is equal to -0.00108263 of the order of 10^{-3}. All other harmonic coefficients of the geopotential are of the order of 10^{-6} and less. Often the zonal harmonic coefficients are denoted by $J_n = -C_{n,0}$, so [2]:

$$V = \frac{fM}{r}\left[1 - \sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n J_n P_{n,0}\left(\sin\Phi\right) + \right.$$
$$\left. + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{a_e}{r}\right)^n \left(C_{nm}\cos mL + S_{nm}\sin mL\right) P_{nm}\left(\sin\Phi\right)\right] \qquad (3)$$

In this equation, the largest harmonic coefficient J_2 has a plus sign.

Due to the symmetry of the Normal Earth with respect to its axis of rotation and the plane of the equator, in the representation of the normal potential of attraction V_0 in a number of spherical functions all non-zonal and odd zonal terms will be absent. The result is [2]:

$$V_0 = \frac{fM_0}{r}\left[1 - \sum_{n=1}^{\infty}\left(\frac{a_e}{r}\right)^{2n} J_{2n}^{0} P_{2n,0}(\sin\Phi)\right] \qquad (4)$$

Proceeding from this equation, two more conditions arise when choosing the parameters for Normal Earth:

3) The mass of the Normal Earth must be equal to the mass of the real Earth, i.e

$$fM_0 = fM \qquad (5)$$

4) Zonal harmonic coefficients of the geopotential of the second order for the Normal Earth and the real Earth must match:

$$J_2^0 = J_2 \qquad (6)$$

One more condition can be written for the selection of Normal Earth:

5) The normal potential of the force of gravity on the surface of the Normal Earth U_0, must be equal to the real potential of the force of gravity W_0 of the geoid.

To fully describe the shape of the Normal Earth and the external gravitational field created by it (or the external gravity field), it is necessary and sufficient to know the parameters fM_0, U_0, J_2^0 and ω. Practically proceeding both from this idea and from the representation (1) of the drag potential as a series of spherical functions, it is preferred instead to include U_0 as an output constant describing the Earth Normal, the equatorial radius a_e. In this regard, condition 5) is formulated as follows:

5') the equatorial radius of the Earth Normal should be chosen in such a way that the volume of the Earth Normal is equal to the volume of the geoid.

When condition 5') is fulfilled, the insignificant differences between the geoid and the quasi-geoid, as well as the dependence of the shape of the quasi-geoid on the selected system of normal heights, are ignored, and instead of the geoid, the quasi-geoid is used in the derivation of a_e. In the future, when these conditions become essential, it will be necessary to use the more precise condition 5) [2].

Conclusions

If the Earth were a "liquid" rotating planet, then to determine its shape it would be sufficient to know the value of the external potential of the force of gravity; then the Earth's surface would be a smooth surface whose equation is determined by the external potential of the planet's surface [3].

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